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Evolution of Modern Battle: An Analysis of Historical Data

A Monograph

by

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ABSTRACT

EVOLUTION OF MODERN BATTLE: AN ANALYSIS OF HISTORICAL DATA by MAJ Charles D. Allen, USA, 85 pages.

This monograph conducts an analysis of historical data to detect trends in war and the evolution to modern tactical battle. It also investigates the continued use of Lanchester-based attrition models.

Military analysts have used the attrition models of Lanchester in an attempt to capture the dynamics of modern battle to answer questions for tactical and strategic decisionmakers. These questions involve technical evaluations of weapon systems, force structure, doctrinal issues, and training.

The methodology uses historical data of 1080 tactical land battles compiled in a study of battles from the years 1618-1905. The data was partitioned into two sets, pre- and post-American Civil War, to test the hypothesis that the conduct of warfare changes around 1865. The evaluation used a generalized form of Lanchester's equations in the form of a linear regression model to indicate which Lanchester-based attrition model best represented the data. A standard statistical hypothesis test called the Student "t" test was applied to determine if the population parameters of the data sets were statistically different and indicated a change in the conduct of battle.

The quantitative analysis revealed that Lanchester's Linear and Square Laws do not adequately describe the battles of this study. This implies that attrition process and battle conditions postulated by these laws are not supported. Peterson's Logarithmic Law is more representative of the data and implies that the casualty producing power of a force increases as the force size decreases.

The evolution of modern battle is marked by quantitative difference as measured by the casualty to initial force ratio. There is a trend toward decreasing ratios that coincides with the post-American Civil War period. The five technological innovations of the nineteenth century: rifled muskets, breechloaders, magazines, barbed wire, and smokeless powder were the catalysts of change in the tactical conduct of warfare. The lethality of the new weapons with their increased accuracy and range combined with the substantial increase in rates of fire forced the change in battlefield tactics.

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I. <u>Introduction</u>

<u>Background</u>

Military analysts have used the attrition models of Lanchester in an attempt to capture the dynamics of modern battle to answer questions for tactical and strategic decisionmakers. These questions involve technical evaluations of weapon systems, force structure, doctrinal issues, and training.

Validation studies have been conducted on the fundamental Lanchester equations with mixed results. James Schneider discussed the shortcomings of the Lanchester equations when applied to historical data in his paper, "The Exponential Decay of Armies in Battle." A recent analysis of data from the National Training Center by Major Ron Johnson failed to validate the Square Law. The analysis of historical data in these studies implies that the Logarithmic Law proposed by Peterson seems more appropriate in reflecting casualties incurred on the modern battlefield.

Both authors suggest that the dynamics of the modern battlefield are too complex to be captured in the Lanchester attrition models. However, the Department of the Army is currently using computer simulations and war games based largely upon the Lanchester equations that have shown limited validity for historical data.

Significance of Military Models

Military professionals throughout the ages have been concerned with developing the best force structure, best equipment, and the best tactics to use in defeating their enemies. Frederick the Great and Napoleon were both commanders who conducted war games and maneuvers as a tool to train their armies.[3] Throughout the nineteenth and twentieth centuries, military commanders have used maneuvers to test out new ideas on tactics and the use of technology. These exercises were models used to validate concepts to be employed in actual combat.

Models are, to varying degrees, an attempt to mimic real events and conditions. They are an "abstract representation which is used for the purpose of prediction and to develop understanding about the real-world process."[illegible] The U.S. military has used models to represent friendly and enemy forces in conflict. In addition to the traditional troop maneuvers of REFORGER and Team Spirit, the Department of Defense has incorporated models through the use of computer simulations and interactive war games into its defense planning and training.

The Department of the Army relies heavily on the use of military models to conduct analysis on tactical engagements from task force to corps level. Models, such as FORCEM, are employed to evaluate the potential

effect of new weapon systems, the application of doctrine, and the effect of force structure changes from task force through theater level. The FIRST BATTLE military model is used to train battalion through corps-level commanders and staffs in the actions required to effectively manage resources and fight their forces.

Table A provides a listing of the computer models used most often used by TRADOC Analysis Command (TRAC). These models are currently in use by the Combined Arms Center (CAC) Combat Development and Training Development Directorates.

Table A - TRADOC Most Often Used Models

MODEL	USE	UNIT LEVEL	ATTRITION	VALIDATION
ACABUG	Analysis	Task Force	One-on-one	No
AMDC	Analysis	Task Force	One-on-one	No
ARTBASS	Training	Task Force	Lanchester	No
BABAS	Training	Battalion	One-on-one	No
BFS	Decisions	Brigade	Lanchester	On-going
CASTFOREM	Analysis	Task Force	One-on one	Yes
CBS(JESS)	Training	Corps/Div	Lanchester	On-going
COMBAT-SIM	Training	Task Force	One-on-one	On-going
CORBAN	Training	Corps/Div	Lanchester	On-going
FIRST BATTLE	Training	Bn/Corps	Lanchester	No
FORCEM	Analysis	Theater	Lanchester	No
JANUS(T)	Training	Task Force	One-on-one	No
JANUS(L)	Analysis	Task Force	One-on-one	No
JTLS	Training	Theater	Lanchester	No
SOTACA	Training	Theater	Lanchester	No

The "real-world" process of forces in combat involves the destruction of enemy units while receiving

friendly casualties. For a model to be useful it must reflect the destructive process of force attrition. The "engine" of a simulation model is based on a mathematical expression of the attrition process. Table A presents the two types of attrition methods employed to decrement forces, one-on-one, and Lanchester-based methods. One-on-one attrition models calculate attrition based on probabilities of target acquisition, target hit, and target kill given a hit for each weapon and target in the engagement. Lanchester-based attrition models, which will be discussed in greater detail in this paper, are focused on the rate of destruction of the force over time based on force characteristics of initial strength level and fighting effectiveness.

In order for a model to be credible in its representation of reality, it must undergo a validation process. TRAC defines model validation as "a formal process of ensuring that the model's behavior is in agreement with the real world experiences and that it is comparable with other more valid model outcomes. The aim of model validation is to increase the degree of confidence that the events inferred by the model will occur."

Of the models listed in Table A only CASTFOREM has been considered validated (because it agreed with

another established model). The remaining models either have not been validated or the validation process is on-going. The use of models that have not been validated should raise questions on the acceptability of their results for defense planning and training purposes.

In order to properly validate a model, its outcomes should be compared to historical data. COL (Ret) Trevor Dupuy writes that:

> If we can collect enough reliable data from military data from military history we should be able to determine patterns of conduct, performance, and outcomes that will provide basic insights into the nature of armed conflict, and that will indicate trends to assist military planning for the future.

The purpose of this monograph is twofold. The first is to determine if the analysis of historic data reflects trends in war and the evolution to modern tactical battle. The second purpose is to investigate whether the continued use of Lanchester-based attrition models is warranted based on the results of the analysis.

The methodology will use historical data of tactical battles compiled in a study of 1500 battles from the years 1618-1905. The data will be partitioned into two sets, pre- and post-American Civil War, to test the hypothesis that the conduct of warfare changes

around 1865. The evaluation criteria will use a generalized form of Lanchester's combat attrition equations in the form of a linear regression. The regression model will indicate which Lanchester-based attrition model best represents the data sets. A standard statistical hypothesis test called the Student "t" test will be applied to compare population parameters of the data sets. The test will determine if the parameters are statistically different and can indicate potential change in the conduct of warfare.

Assumptions

For the purpose of the monograph, only land battles will be the used from the historical study. The investigation of attrition models will focus on deterministic models, those whose parameters do not vary over time, and the models that apply to homogeneous forces. The term modern battle will refer to battlefield conditions as they occurred after the American Civil War.

II. Study of Attrition Models

Background on Lanchester

Frederick William Lanchester (1878-1946) was a brilliant English mathematician, engineer, and inventor. He was highly respected for his work in aerodynamics, economics, and military strategy. Lanchester was an early advocate for the use of aircraft, and he envisioned the great impact that aircraft would have on the conduct of war. Beginning in 1914 he published a number of articles that attempted to quantify that impact.

He developed a mathematical analysis, known as the Lanchester equations, to predict the outcome of military engagements. His intent was to demonstrate the principle of concentration of firepower for land combat and then apply it to the conditions of air combat.

Modern usage has adopted and modified Lanchester's basic equations to model attrition for land forces. Lanchester acknowledged the argument that ground combat had "so many unknown factors such as morale or leadership of the men, the unattended merits or demerits of weapons, and the still more unknown 'chances of war,' [that] it is ridiculous to pretend to calculate anything." However, Lanchester argued that any analysis of battle would always begin with a counting of forces available to each side.

Development of Lanchester's Square Law

In his original work Lanchester described the conditions of ancient and modern warfare. In ancient warfare, he contended that battles were characterized by individuals engaged in combat, man against man. The battle consisted of approximately equal forces at the point of contact. In modern warfare, combat is a collective activity where units fight against units and weapon systems are employed against weapon systems. The survival of a force is based on the destruction of the enemy.

This view of modern warfare reflects the progress of technology in Lanchester's time with the introduction of long-ranged, magazine-fed rifled weapons. In modern battle, the "concentration of superior numbers gives an immediate superiority in the active combat ranks...[where a]...numerically inferior force finds itself under far heavier fire ...than it is able to return."

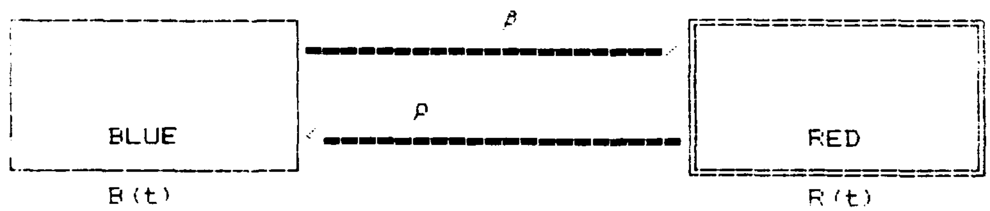

Figure 1 - Lanchester's Model of Combat

The combat dynamics modeled by Lanchester reflects the interaction between forces as shown in Figure 1. The Lanchester equations 1 and 2 below result in Lanchester's Square Law (equation 3). The force level at any time t for the Blue or Red unit is represented by B(t) and R(t), respectively. The initial force strengths are represented by B_o and R_o. The rate at which Blue destroys Red per unit time is given by β. The rate at which Red destroys Blue is given by ρ. These parameters, β and ρ are known as the attrition rate coefficients. If β and ρ were equal, then the Square Law would imply that victory would go to the force with the greatest initial strength.

Lanchester's Equations

EQN 1	EQN 2
$dB/dt = -\rho R$ for $B>0$	$dR/dt = -\beta B$ for $R>0$
where $dB/dt = 0$ for $B=0$	where $dR/dt = 0$ for $R=0$

Square Law

EQN 3 $$\beta(B_o^2 - B^2) = \rho(R_o^2 - R^2)$$

There are five basic assumption for the application of the Square Law to modern combat. Those assumptions are:

1. Two forces are engaged in combat. The forces are homogeneous in composition (i.e. weapons systems are essentially similar, if not identical).

2. Each force is in range of the opponent's weapons.

3. Attrition rates are known and do not vary over the course of the battle (this is the condition for a deterministic model).

4. Exact enemy locations are known and battle damage assessment allows shifting to new targets.

5. Weapons fire is evenly distributed over surviving targets.

Current application of the Square Law in simulations is used to represent conditions where the opposing forces employ "aimed" fire. Under this scenario the time to acquire targets is assumed to be constant and independent of the number of available targets. The Square Law is also used to represent conditions where the opposing forces use "area" fire and occupy a constant-density defense.[1]

Aimed fire assumed that "each firer on the red side will pick targets on the blue side and try to kill it." This generally applies to the use of direct-fire weapons--rifle, tank, antitank, and precision guided munitions.

Area fire is assumed where the individual firer is not able to acquire a specific target but fires into the general location of the targets. This generally applies to the use of indirect fire weapons, such as artillery,

along with automatic weapons and other direct fire weapons used against a position.

Development of Lanchester's Linear Law

Another condition of modern warfare was characterized by Lanchester as "firing into the brown" where volley firing was employed against an enemy position.[16] Lanchester acknowledged that such firing was more common in land combat than in air and sea engagements under conditions where ground forces are sometimes unable to acquire and engage specific targets. The Linear Law is derived from the following equations 4 and 5.

EQN 4	EQN 5
$dB/dt = -\rho BR$ for $B,R>0$	$dR/dt = -\beta RB$ for $R,B>0$
where $dB/dt = 0$ for $B=0$	where $dR/dt = 0$ for $R=0$

These equations have included an additional variable of the targeted unit's strength. The attrition process is now a function of the number of firers, the number of targets, and the rate at which the firers kill the targets. The solution of the above equations leads to the Linear Law of equation 6.

Linear Law

EQN 6 $$\beta(B_o - B) = \rho(Ro - R)$$

The Linear Law requires a change to assumptions 4 and 5 for the Square Law. The new assumptions are[15]:

4a. Information is only available on the general enemy location.

5a. Units are evenly distributed over a general area.

Current application of the Linear Law is directed in representing circumstances where opposing sides use area fire and a constant-area defense. A second circumstance appropriate to the use the Linear Law occurs where both forces use aimed fire with the rate of target acquisition inversely proportional to the number of targets (i.e., the more targets to choose from, the longer the time required to select and engage an individual target):

Shortcomings of Lanchester's Models

Lanchester's original models have been critiqued as overly simplifying the conduct of battle. The deciding factor in determining victory in battle is based greatly on the force ratio of the opponents and the firepower that is reflected in the attrition rate coefficients. Each of the five basic assumptions can be attacked based on the actual conditions of warfare.

1) Opposing forces are rarely homogeneous. Differing levels of technology can produce weapons which have dissimilar capabilities in the era of modern warfare.

2) Because of the dispersion of the modern battlefield all forces will not be within weapons range of the other.

3) Attrition rates are not constant over time and vary over the duration of the battle.

4) The fog of war does not always allow the detection of specific and/or general enemy locations during battle. Battle damage assessment is not available to the extent where multiple hits on a target can be prevented.

5) The friction of war and non-linearity of the battlefield does not accommodate the idea of even distribution of fires or distribution of forces.

There are other combat dynamics that are not characterized in the Lanchester models. They are the effect of morale, leadership, training and technology on the performance of soldiers on the battlefield.

<u>Adaptation of Lanchester's Equations by Brackney</u>

Harold Brackney disagreed with the concept that combat between forces was purely aimed fire or area fire engagements. He viewed battle as a contest between two forces-- an attacker and a defender. His model of combat represented by equations 7 and 8 reflects a combination of conditions for the Square and Linear Laws.

Brackney's Equations

EQN 7	EQN 8
$dB/dt = -\rho R$ for $B>0$	$dR/dt = -\beta RB$ for $R>0$
where $dB/dt = 0$ for $B=0$	where $dR/dt = 0$ for $R,B=0$

The Blue attacker is unable to acquire a specific Red defender. As a result, the Blue firer aims and fires in the direction of the defender's position using area fire. The Red defender, possessing the advantages of the defense, is able to select and place aimed fire upon specific Blue attackers. The same concept and model has been applied to guerilla warfare where the Red force represents the insurgents and the Blue force represents the government forces. The attacking guerillas in ambush operations would fire upon government soldiers from covered and concealed positions. The surprised forces would not be able to identify specific targets but would place area fire in the general direction of the attackers. The Brackney Mixed Law is given by equation 9.

Mixed Law

EQN 9	$(\beta/2)(B_o^2 - B^2) = \rho(R_o - R)$

The same assumptions used for Lanchester's Square and Linear Law are applicable to the Mixed Law.

<u>Adaptation of Lanchester Equations by Peterson</u>

Another adaptation to the basic Lanchester equations was made by Richard H. Peterson. He reasoned

that as the unit size increases, a greater number of targets are presented to the opponent. Therefore, the attrition of a force is a function of the force level and the attrition rate inflicted by the opponent as reflected in equations 10 and 11. Peterson held that this attrition process only applied in the early stages of battle. Peterson contended that the "force incurs losses that are proportional to its own size and [are] largely independent of the enemy's numbers."[22]

Peterson's Equations[22]

EQN 10	EQN 11
$dB/dt = -\rho B$ for $B>0$	$dR/dt = -\beta R$ for $R>0$
where $dB/dt = 0$ for $B=0$	where $dR/dt = 0$ for $R=0$

The solution of Peterson's equations results in the Logarithmic Law.

Logarithmic Law

EQN 12 $$\beta \text{ Log } B_o/B = \rho \text{ Log } R_o/R$$

The key assumption for the Logarithmic Law is all weapons are within the range of the opponent but no two weapons are intervisible. This condition could exist through the use of cover, concealment, and/or camouflage of the targets.

Development of Willard's Linear Regression Model

The first comprehensive attempt to test Lanchester's equations versus historical data was conducted by Willard and published in 1962. Willard's stated intent was to determine if the Square or Linear Laws "adequately describes the course and outcome of battles between armies." To conduct the test, Willard generalized the basic Lanchester equations (1 ,2 ,4 and 5) to make them independent of time. Using mathematical approximations and substitution, he arrived at the following model.

Willard's Linear Regression Model

EQN 13	$\log B_c/R_c = \log E + \gamma \log R_o/B_o$

This equation reflects a linear relationship between the ratio of casualties and the ratio of initial force levels. This is a predictive model where the independent variable is the logarithm of the initial force ratio, R_o/B_o. The casualties inflicted on the Blue and Red forces are depicted by B_c and R_c, respectively. E represents the exchange ratio between the forces. The parameter, γ, indicates which attrition model is most representative of the data.

Table B - Parameter Values

γ	Appropriate Law
+1	Square Law
.5	Mixed Law
0	Linear Law
−1	Logarithmic Law

Results of Validation Attempts

As stated earlier, models are designed to represent reality and are used as tools of prediction. Validation of a model is the process of comparing the results of the model to data collected for the reality that it attempts to represent. The purpose of model validation is to provide a degree of confidence that the results predicted by the model will occur.

The number of studies that have tested the Lanchester models is limited. Researchers have claimed that the amount and quality of data collected from conflicts of the twentieth century are not readily available to test the models.[illegible] Hence, the number of cases that claim validation against observed data are very rare.

The validation of the Lanchester-based attrition models have met with mixed success. Both James Schneider and Ronald Johnson have provided a brief summary of the more prominent attempts to validate the

Square and Linear Laws. Table C presents a survey of validation attempts and their results.

Table C - Survey of Validation Studies

Name	Data Source	Result
Busse	Inchon-Seoul Campaign	Unable to validate the Square Law
Engel	Battle of Iwo Jima	Questionable validation of the Square Law
Fain	60 land battles of World War II	Suggestion of Logarithmic Law
Johnson	National Training Center	Suggestion of Logarithmic Law
Schmieman	Bodart's *Lexicon*	Square and Linear Laws inadequate
Willard	Bodart's *Lexicon*	Square and Linear Laws "poor choices"

The most comprehensive test of the attrition models was conducted by Daniel Willard. Willard assumed the validity of both the Square and Linear Laws and expected that the analysis of historical data would reflect a combination of the use of area and aimed fire. He projected that performing a regression analysis using equation 13 would result in γ approximately equal to .5. This would indicate that the Mixed Law best represented the data. His analysis yielded γ parameter values ranging from -.27 to -.87 with a average near -.5. This result refutes Lanchester's basic equations

in favor of Peterson's Logarithmic Law. The implication is that the casualty-producing power of a force increases as the force size decreases. In other words, a smaller unit presents less targets to its opponent and is more efficient in its ability to inflict casualties. This is counter to Lanchester's basic premise that the number of casualties incurred is inversely proportional to the initial force ratio. Lanchester posited that an increased force ratio would reduce casualties for the superior force.

III. Historical Battles as a Data Source

The body of data used for the analysis is based on the information complied in Gaston Bodart's *Militaerhistorisches Kriegs-Lexicon.* Bodart's study covers the 1,493 major engagements that occurred between the years 1618 to 1905. The period begins with the Thirty Year's War, progresses through the Napoleonic Wars, the American Civil War, and concludes with Russo-Japanese War of 1905.

The *Lexicon* is accepted as an accurate and comprehensive source of data on engagements for the period. The information collected by Bodart includes the identification of combatants, location of battle, initial strengths of the forces, and the casualties sustained.

The data is organized in chronological order. Appendix A is a sample page from the text. Bodart categorized the engagements into eight classes. The classes of *treffen*, *gefecht* and *schlact* can be equated to meeting engagements. The remaining classes of *belagerung*, *einnahme*, *erstuermung*, *kapitulation*, and *ueberfall* are attacks against fortified positions.

The analysis was conducted using data on 1080 land battles from the period of the study. The data set was extracted from an annex of Schmieman's dissertation. It consists of the year, initial force strengths for Red and Blue, and the casualties incurred by each side (Appendix C).

IV. Statistical Analysis of Historical Data

The research question is to determine if the quantitative analysis of historical data reflects trends in the conduct of battle. In specific, the analysis was conducted to identify trends in the data that would mark the evolution of modern battle beginning with the American Civil War.

William Schmieman's analysis showed that categorizing the data by battle size and percent casualties yielded results that generally were not statistically different. My analysis initially partitioned the data based upon the years of occurrence, pre- and post-Civil War era.

The preliminary analysis was conducted utilizing Willard's linear regression model. As previously presented, the model is:

$$\text{EQN 13} \qquad \log B_e/R_e = \log E + \gamma \log R_o/B_o$$

The initial regression was performed with the 1080 battles to determine which Lanchester-based model best represented the data as indicated by the value of γ. The arbitrary assignment of Red to the winning force in Bodart's data was done to maintain consistency in the analysis process. The results of the regression are provided in Appendix B-1. The value $\gamma = -.475$ is

consistent with Willard's analysis where the values ranged from -.27 to -.87.[38] Referring back to Table B (page 17), this suggests that Peterson's Logarithmic Law represents the historical data better than the Square or Linear Laws.

To evaluate the adequacy of the regression model in describing the data, a test of the null hypothesis is required. A null hypothesis is a statistical challenge to the value of the parameters determined by the regression. The regression parameters are the slope of the regression line, in this case γ, and the intercept point along the vertical axis, Log E. The null hypotheses posit that the parameters, γ and Log E, are both equal to zero. This would indicate that a linear relationship does not exist for the data.

A Student "t" test is used to evaluate the hypothesis at a specified confidence level. A confidence level is a statistical statement of the frequency that the test results will reflect the actual conditions. Hence, a 95% confidence level asserts that the results of the hypothesis test will reflect the true state 95 times out of a 100. A computed "t" statistic is calculated from the population parameters of the mean, sample variance, and number of observations. It is compared to the t-value associated with a specific confidence level. If the computed "t" exceeds the

t-value, the null hypothesis is rejected.

The null hypotheses, Log E = 0 and $\gamma = 0$, were tested at the 95% confidence level. Since the computed "t" statistics both exceed the critical value of $t_{.05}$ = 1.96, the null hypotheses were rejected.

Another indicator of the adequacy of the model is the coefficient of determination, R-square. R-square is calculated from the degree of correlation between the independent and dependent variables of the regression. The value of R-square indicates the amount of variability in the data that is accounted for by the regression. For this model, the R-square value shows that the model only accounts for 15 percent of the variability in the data. This value does not indicate a very good fit of the data to the model.

The results of the regression analysis of the data when partitioned into pre-1861 and post-1861 are provided in Appendix B-2 and B-3, respectively. The pre-1861 model suggests that the Logarithmic Law is appropriate with $\gamma = -.475$. The null hypotheses were rejected at the 95% confidence level. The R-square value indicates that 15 percent of the variability in the data is accounted for in the model. The post-1861 results are similar. With $\gamma = -.496$, the Logarithmic Law is a better representation of the data. Again, the null hypotheses were rejected at the 95% confidence

level. There was a slight improvement in the R-square value to 18 percent of the variation in the data explained by the model.

Table D - Summary of Regression Results

Period	Log E	y	R-square
1620-1905	.652	-.475	15%
Pre- 1861	.700	-.475	15%
Post-1861	.380	-.496	18%

Although the regressions indicate a linear relationship between the logarithm of the initial force ratio and the logarithm of the casualty ratio, the low R-square values do not inspire a "degree of confidence in the outcome." The regressions clearly show that the basic Lanchester equations as manifested in the Square and Linear Laws are not supported through the analysis of historical data.

In an attempt to detect trends in the data, the ratio of casualties to initial strengths, R_c/R_o and B_c/B_o was investigated. The logarithmic function was not used since it transforms the data and the original magnitude is lost. A factor of ten difference in the original data becomes a factor of two when the logarithm function is applied. For example, Log .1 = -2.303 and Log .01 = -4.605. This characteristic is referred to as

"log compression." The use of transformed, compressed data hinders the analysis of the population parameters.

Table E - Casualty Statistics (Percentages)

	Red	Blue
mean	.0858	.1759
st dev	.083	.153

The average casualties sustained by the victor over the 1620-1905 period was 8.6 percent, while the loser suffered 17.6 percent casualties. A paired "t" test was conducted at the 95% confidence level to test the hypothesis that the two means were equal. The computed statistic $t = 20.35$ greatly exceeded $t_{.05} = 1.96$ and implies that the means are statistically different. This finding that percent losses of the victorious force are generally less than the losing force is consistent with the analysis conducted by Richard Helmbold on a sample of ninety-two battles.

James Schneider proposes in an article that a change in the conduct of warfare occurred to give "rise to [a] qualitatively distinct style of military art." He posits that the American Civil War was the event that marked the transition from the classical Napoleonic style of military art to operational art. A "t" test was performed with the ratios partitioned into pre- and

post-1861 data sets to test Schneider's hypothesis.

Table F - Statistics of Partitioned Data

	Red		Blue	
	pre-1861	post-1861	pre-1861	post-1861
mean	.0864	.0826	.1843	.1271
variance	.007	.006	.024	.011
"t"	-.53		-5.82	

The analysis of the victor's casualties to initial strength ratio implies that there is no statistical difference between the pre- and post-1861 means at the 95% confidence level. The percent of Red casualties remained consistent over the two periods.

The analysis of the Blue loser yielded different results. There is a statistical difference at the 95% confidence level in the means of the percent casualties between the pre- and post-1861 data sets. The average percent casualties dropped from 18.4 percent of the initial strength to 12.7 percent.

Further analysis was conducted by aggregating both the Red and Blue data and performing a "t" test on the composite data with following results:

Table G - Statistics of Aggregated Data

	pre-1861	post-1861
mean	.1354	.1048
variance	.0186	.0087
"t"	3.86	

The hypothesis that the means of the pre- and post-1861 casualty to initial force ratio are equal was rejected at the 95% confidence level. This implies that there is a statistical difference in the pre- and post-1861 means.

A linear regression of the complete data set was performed using the model:

EQN 14	$Rc/Ro = \epsilon + \alpha(Bc/Bo)$

where ϵ is the y-intercept and α is the slope of the regression line. The results of the regression are provided in Appendix B-4. At the 95% confidence level, the null hypotheses, $\epsilon = 0$ and $\alpha = 0$, were both rejected with $t_{.05} = 1.96$. The coefficient of determination, R-square, indicates that only 13% of the variability in data is explained by the model.

The results of the regression analysis of the data when partitioned into pre-1861 and post-1861 are

provided in Appendix B-5 and B-6, respectively. In the analysis of the pre-1861 data, the null hypotheses were rejected at the 95% confidence level. The R-square value indicates that 12 percent of the variability in the data is accounted for in the model. The resulting regression coefficients for post-1861 are similar. Again, the null hypotheses were rejected at the 95% confidence level. However, there is a significant improvement in the R-square value. In this case 34 percent of the variation in the data explained by the model. This indicates a greater correlation (approximately 58 percent) between the casualty to force ratio for the Red and Blue forces.

Table H - Summary of Regression Results

Period	ϵ	α	R-square
1620-1905	.051	.197	13%
Pre- 1861	.053	.183	12%
Post-1861	.030	.419	34%

The data was further partitioned into a total for four periods: 1620 - 1800, 1800 - 1861, 1861 - 1865, 1865 - 1905. The results of the regression analysis are provided in Appendix B-7 through B-10. It is interesting to note that the analysis indicates a linear relationship exists between the casualty to force ratios for Red and Blue. That relationship as measured by

R-square is generally improving through the Civil War and post-Civil war periods.

Another set of "t" tests were performed on the aggregated means (both Red and Blue) over the sub-periods. Table I is a summary of the descriptive statistics for the partitioned data.

Table I - Summary of Percent Casualties Ratio

Period	Mean	Variance	Number of Observations
1620-1800	.1465	.0218	1136
1800-1861	.1175	.0130	704
1861-1865	.1152	.0074	98
1865-1905	.1003	.0092	222

Table J provides the results of the "t" test between the means of the partitioned data set. There is a statistical difference at the 95% confidence level where $t_{.05}$ = 1.96 for the following cases: pre-1800 and 1800-1861, pre-1800 and 1865-1905, 1800-1861 and 1865-1905.

Table J - Computed "t" values

Period	1620-1800	1800-1861	1861-1865	1865-1905
1620-1800	--	--	--	--
1800-1861	4.46	--	--	--
1861-1865	1.90	.20	--	--
1865-1900	4.49	2.04	1.32	--

V. Evaluation of Results

Qualitative Analysis of Results

The quantitative analysis of the data is summarized as follows:

1. Lanchester's Linear and Square Laws do not adequately describe the 1080 battles of this study. This implies that battle conditions postulated by these laws are not supported. The Square Law maintains that casualties sustained by a force will be inversely proportional to the size of the force. The Linear Law predicts that the attrition process is largely independent of the size of the forces engaged but is solely a function of the attrition rate coefficients for each force.

2. Peterson's Logarithmic Law is more representative of the data and implies that the casualty producing power of a force increases as the force size decreases. An extension of this assertion is that larger units are less efficient in their attrition of the enemy than are smaller units.

3. The generalized form of Lanchester's equations represented by Willard's regression model depicts a linear relationship between the logarithm of the casualty ratio and the logarithm of the initial force ratio. However, the relatively low values of R-square indicate that other factors may exists that

better explain the variations in the data.

4. There is a quantitative difference in the conduct of battle as measured by the casualty to initial force ratio for the pre- and post-Civil War periods. There is a general trend toward decreasing ratios over the time period studied. This result supports Schneider's assertion that the conduct of warfare significantly changed with the Civil War. The analysis also shows that the victor sustains fewer percent casualties than the battle's loser.

5. A stronger linear relationship exists using casualty to initial force ratio than is present with Willard's regression model as measured by the coefficient of determination.

The analysis of the historical data reveals that there is a measurable, quantifiable change in the tactical conduct of battle that occurs after the American Civil War. The next step is the qualitative investigation of conditions that existed in the post-1861 period that resulted in the quantitative change.

Schneider discusses the declining casualty ratios in his article, "The Theory of the Empty Battlefield." He posits that "armies incur fewer casualties in the face of more lethal weapons." The weapons' increased lethality is attributable to five technological

innovations of the nineteenth century: rifled muskets, breech-loading rifles, magazines, barbed wire, and smokeless powder.[36] Other innovations that follow as a direct result are the introduction of the machinegun and breech-loading, rifled artillery These inventions had a significant impact on the conduct of battle after their adoption by the modern armies.

Smoothbore muzzleloader dominated the weapons used by armies prior to the mid-nineteenth century. The early development of rifled musket in the 1500's did not lead to their rapid adoption due the technical problems of loading. The rifle became practical with the development of the cylindro-conodial bullet by Captain Norton and its subsequent refinement by Captain Minie in the 1840's.[37] The new bullet could be dropped down the muzzle of the weapon and did not require ramming.

When the musket was fired, the rifling forced the bullet to spin as it travelled down the barrel. The muzzle velocity provided additional range and the spin provided stability which improved the accuracy. A German study in 1840 proposed that the effectiveness of troops armed with rifles increased by three and a half times over those with smoothbore muskets.[38]

The adoption of rifles as practical infantry weapons began in the 1840's with the United States. The British replaced their traditional Brown Bess musket

with the rifle in 1842.[39] The French overcame their bureaucratic reluctance and adopted the Tige rifle for its infantry in 1857.[40]

The first significant use of rifles in combat occurred in the Crimean War where Russian forces armed with smoothbore muskets fought against the British and French forces equipped with rifles. Additionally, the standard weapon of the American Civil War after 1862 was the Springfield, then the Enfield rifle.[41]

The next innovation was the bolt-action breechloader patented in 1835 by von Dreyse. Von Dreyse's needlegun was adopted by the Prussian army in 1840. It had over twice the potential rate of fire as the rifle loaded with a Minie ball. However, the increased rate of fire was offset by the loss of muzzle velocity which limited the range to approximately 600 yards.[42]

The above two innovations had a substantial effect on changing tactics of the post-1861 era. Previous use of smoothbore muskets, with a range of 200 yards, required the use of close order formations to concentrate firepower.[43] Regiments would approach on a narrow front with companies in columns and then battalions would deploy in line and employ volley firing. These tactics resulted in higher casualties due to the increased lethality of the weapons.[44]

During the Civil War, Dennis Hart Mahan observed that "[t]he great destruction of life, in open assaults, by columns exposed within so long a range, must give additional value to entrenched field of battle."[43] Union and Confederate military leaders recognized that the rifle strengthened entrenchments and that defense had gained the power to devastate forces conducting frontal assaults. Subsequently, the use of entrenchments was incorporated into both offensive and defensive operations while the use of frontal assaults was strongly discouraged.[44]

The use of breechloaders allowed the soldiers to fire and reload their weapons while in the prone position thereby reducing their vulnerability to enemy fire. The advantage was clearly illustrated in the Austro-Prussian War of 1866. The Prussians were able to fire at a more rapid rate from prone positions against the Austria who had to stand to reload. The Prussians, with their new weapons and tactics, were able to achieve a six to one firepower advantage over the Austrians.

This lesson was not lost on the French who quickly developed and fielded the *chassepot* breechloader in 1870. The range of the *chassepot* was 1200 yards--more than twice the range of the Prussian needlegun. The increased lethality of the weapons forced a modification

in French tactics where they adopted the use of rifle pits against the Prussian infantry attacks.[48] In turn, the Prussians adopted the American system of advancing one-half of the force in rushes while the remaining half provided covering fires.

The magazine for rifles was initially patented in 1849 in America and the vertical magazine was patented in 1879.[51] The magazine, combined with the other innovations, increased the potential rate of fire of each soldier. Soldiers did not have to reload after each firing and their accuracy could improve since the selection of targets was not interrupted by the need to frequently reload.

An extension of the magazine concept was the development of rapid-fire weapons. The French developed and employed the *mitrailleuse* in the Franco-Prussian War. This machinegun, which fired thirty-seven rifled barrels simultaneously, and the American Gatling gun represented a great increase in firepower produced by a single weapon.

The Russo-Japanese War of 1905 demonstrated the devastating impact of rifles and machineguns when integrated into the defense of fortified positions. Japanese frontal assaults against Russian fortifications yielded exceptionally heavy casualties.

The introduction of barbed wire on the battlefield

served to greatly enhance the defense. Barbed wire severely restricted the ability of attacking forces to breach entrenchments. Attempts to get through the wire increased the amount of time that the attackers were exposed to the rifle and machinegun fires. Tactics employing frontal assaults were very costly in the Russo-Japanese War. "[A]lways the assaults ended up with the Japanese hung up and massacred on the barbed wire..."

The last innovation to impact on the evolution of modern battle was smokeless powder. The first impact was the increase in muzzle velocity and a greater accuracy. The second impact provided a new concern to the soldiers on the modern battlefield. Prior to its introduction, the use of firearms created a signature cloud of smoke and provided the location of the weapon on the battlefield. The enemy would be able to fire more shots prior to being detected. The smokeless powder presented the additional specter of the invisible enemy. This compounded the psychological stress already experienced by the combatants.

The impact of magazine rifles and smokeless powder was demonstrated in the Boer Wars. The initial use of blackpowder charges for British artillery readily identified their position. It, in turn, subjected them to intense and accurate counterfire. Also, the

increased range and accuracy provided by the new smokeless powder amplified the effectiveness of the Boer marksmen. Accounts of the war relate the participant's feeling of "'being opposed to an invisible foe'"[54] while under the long-range fires.

The introduction of the breech-loading rifle increased the lethality of the weapons on the battlefield and reshaped the tactics of the nineteenth century. The need to reduce vulnerability to the new weapons reestablished the superiority of the defense and led to the adoption of trenches and fortifications. Modern armies had to accept that frontal assaults against entrenchment would result in heavy casualties. The tactics evolved to the use of skirmish order to disperse troop for such assaults and relied more on flanking movements against defenses.

The magazine with the resulting increased rate of fire and smokeless powder allowed the individual soldier to assume a greater sector of responsibility on the battlefield. Smokeless powder also forced the units to disperse for survivability against an invisible enemy and increased the moral demands on the soldiers. Bloch accurately predicted that "[b]efore the introduction of long-ranged firearms, battlefields were no larger than the exercise of modern brigades. The battlefield of the future would prove to be much greater

than those of the past."

The conduct of modern battle evolved in the post-1861 era. The technological innovations presented above were employed during the era and shaped the tactics by which armies fought. The decline in casualty ratios noted in the quantitative analysis can be attributed as the reaction of the armed forces to emerging conditions of warfare in order to protect themselves from the horrors of the new battlefield.

VI. Conclusions and Implications

Summary of Findings

The quantitative analysis reveals that Lanchester's Linear and Square Laws do not adequately describe the 1080 battles of this study. This implies that attrition process and battle conditions postulated by these laws are not supported. Peterson's Logarithmic Law is a more representative of the data and implies that the casualty producing power of a force increases as the force size decreases. An extension of this assertion is that larger units are less efficient in their attrition of the enemy than are smaller units. These findings are consistent with other studies.

The generalized form of Lanchester's equations represented by Willard's regression model depicts a linear relationship between the logarithm of the

casualty ratio and the logarithm of the initial force ratio. However, the relatively low values of R-square indicate that other factors or models may exist that better explain the variations in the data.

Caution must be exercised in applying the lessons derived from the pre-1861 data to modern conditions. The conduct of modern battle was shaped greatly by the technological advances that were not available during that period.

The evolution of modern battle is marked by quantitative difference as measured by the casualty to initial force ratio. There is a general trend toward decreasing ratios over the time period studied that coincides with the post-American Civil War period. The five technological innovations of the nineteenth century: rifled muskets, breechloaders, magazines, barbed wire, and smokeless powder were the catalysts of change in the tactical conduct of warfare. The lethality of the new weapons with their increased accuracy and range combined with the substantial increase in rates of fire forced the change in battlefield tactics.

Implications

The Lanchester equations as manifested in the Square and Linear Law are not representative of historical data. Analysts have persisted in these laws

because of the relative ease of employment in computer simulations and their widespread acceptance. If the generalized form of the Lanchester equations as represented by Willard's model is accepted as capturing the dynamics of battle, then the Logarithmic Law and its implications must also be accepted. If not, analysts should investigate the use of other more representative models.

The acceptance of any model must be tentative until it has been subjected to the validation process. The decisions that are made based upon the model's result impact greatly on force structure, doctrine and the training of our military leadership. If the underlying model is not representative of the realities of battle, the lessons gleaned from the simulations will be equally fictitious.

1870 27./11.

SCHLACHT
bei
Amiens (5.)
(Villers-Bretonneux)

(Stadt in Frankreich, Hauptstadt des Dép. Somme, 133 km nördl. von Paris).

Sieg der **Deutschen** (30.600 Inft., 4.400 Kav., 137 Gesch. = 35.000 M.) unter Gen. d. Inf. Fh. v. Manteuffel über die **Franzosen** (23.500 Inft., 1.500 Kav., 42 Gesch. = 25.000 M.) unter Gen. Faidherbe.

Verluste:

	230 (1 Stb. 18 Offz.)	tot	(1 Stb. 13 Offz.) 260	
	1.070 (6 „ 50 „)	verwundet	(3 „ 33 „) 1.140	
3·7% =	1.300 (7 Stb. 68 Offz.)	**Blutige Einbusse**	(4 Stb. 46 Offz.) 1.400	= 5·6%
0·8% =	300 (— 1 „)	vermißt, gefangen	(— 20 „) 2.100	= 8·4%
4·5% =	1.600 (76 Offz.)	**Gesamt-Verlust**	(70 Offz.) 3.500	= 14·0%

Verl. an Trophäen: 9 Kanonen (= 21%), 2 Fahnen.

1870 15.—27./11.

BELAGERUNG
und
EINNAHME
von
La Fère

(Stadt in Frankreich, Dép. Oise, an der Oise, 22 km nordwestl. von Laon).

Die **Deutschen** (5.000 M.) zwingen die **französische** Besatzung (2.300 M., 70 Gesch.) zur Übergabe. Die Garnison wurde kriegsgefangen.

1870 28./11.

SCHLACHT
bei
Beaune-La-Rolande (4.)

(Stadt in Frankreich, Dép. Loiret, an der Rolande, 19 km südöstl. von Pithiviers).

Deutsche				**Franzosen**
GFM. Pz. Friedrich Karl v. Preußen				Gen. Crouzat
		Streitkräfte:		
	34.000	Infanterie	56.000	
	6.000	Kavallerie	4.000	
174 Gesch.	40.000	**Gesamt-Stärke**	60.000	Gesch. 138

Verluste:

	1.000 (1 Stb. 39 Offz.)	tot und verwundet	(11 Stb. 112 Offz.) 2.200	= 3·7%
	— — —	vermißt, gefangen	— — 1.800	= 3·0%
2·5% =	1.000	**Gesamt-Verlust**	4.000	= 6·7%

1 Geschütz. **Verl. an Trophäen:**

Sample Page from Gaston Bodart's, *Militaer-Historisches Kreigs-Lexicon 1618-1905*. Vienna: C. W. Stern, 1908, p. 562.

ALL YEARS

MULTIPLE LINEAR REGRESSION

Dependent Variable: LOG(Bc/Rc)

Variable	Mean	Parameter Estimate	Standard Error	T for HO: parameter=0
Intercept LOG E		0.652	0.03	22.55
LOG(Ro/Bo)	0.244	-0.475	0.03	-13.69

Source	DF	Sum of Squares	Mean Square	F-Value
Model	1.00	154.42	154.42	187.33
Error	1,077.00	887.77	0.82	
Total	1,078.00	1,042.19		

Dependent Mean	0.54
Root Mean Square Error	0.91
Coefficient of Variation	169.36
R-Square	0.15
Adjusted R-Square	0.15

PRE-1861

MULTIPLE LINEAR REGRESSION

Dependent Variable: LOG(Bc/Rc)

Variable	Mean	Parameter Estimate	Standard Error	T for H0: parameter=0
Intercept LOG E		0.700	0.03	22.05
LOG(Ro/Bo)	0.249	-0.475	0.04	-12.62

Source	DF	Sum of Squares	Mean Square	F-Value
Model	1.00	134.51	134.51	159 .14
Error	917.00	775.10	0.85	
Total	918.00	909.61		

Dependent Mean	0.58
Root Mean Square Error	0.92
Coefficient of Variation	157.99
R-Square	0.15
Adjusted R-Square	0.15

POST-1861

MULTIPLE LINEAR REGRESSION

Dependent Variable: LOG(Bc/Rc)

Variable	Mean	Parameter Estimate	Standard Error	T for H0: parameter =0
Intercept LOG E		0.380	0.06	5.85
LOG(Ro/Bo)	0.215	-0.496	0.08	-5.85

Source	DF	Sum of Squares	Mean Square	F-Value
Model	1.00	21.31	21.31	34 .26
Error	158.00	98.26	0.62	
Total	159.00	119.57		

Dependent Mean	0.27
Root Mean Square Error	0.79
Coefficient of Variation	289.01
R-Square	0.18
Adjusted R-Square	0.17

ALL YEARS

MULTIPLE LINEAR REGRESSION

Dependent Variable: Rc/Ro

Variable	Mean	Parameter Estimate	Standard Error	T for H0: parameter=0
Intercept e		0.051	0.00	14.32
Bc/Bo	0.176	0.197	0.02	12.84

Source	DF	Sum of Squares	Mean Square	F-Value
Model	1.00	0.98	0.98	164 .86
Error	1,078.00	6.44	0.01	
Total	1,079.00	7.42		

Dependent Mean	0.09
Root Mean Square Error	0.08
Coefficient of Variation	90.02
R-Square	0.13
Adjusted R-Square	0.13

PRE-1861

MULTIPLE LINEAR REGRESSION

Dependent Variable: Rc/Ro

Variable	Mean	Parameter Estimate	Standard Error	T for H0: parameter=0
Intercept e		0.053	0.00	13.17
Bc/Bo	0.184	0.183	0.02	11.14

Source	DF	Sum of Squares	Mean Square	F-Value
Model	1.00	0.78	0.78	124.01
Error	918.00	5.75	0.01	
Total	919.00	6.53		

Dependent Mean	0.09
Root Mean Square Error	0.08
Coefficient of Variation	91.64
R-Square	0.12
Adjusted R-Square	0.12

POST-1861

MULTIPLE LINEAR REGRESSION

Dependent Variable: Rc/Ro

Variable	Mean	Parameter Estimate	Standard Error	T for H0: parameter=0
Intercept e		0.030	0.01	3.89
Bc/Bo	0.127	0.419	0.05	9.03

Source	DF	Sum of Squares	Mean Square	F-Value
Model	1.00	0.30	0.30	81.58
Error	158.00	0.59	0.00	
Total	159.00	0.89		

Dependent Mean	0.08
Root Mean Square Error	0.06
Coefficient of Variation	73.70
R-Square	0.34
Adjusted R-Square	0.34

PRE-1800

MULTIPLE LINEAR REGRESSION

Dependent Variable: Rc/Ro

Variable	Mean	Parameter Estimate	Standard Error	T for H0: parameter=0
Intercept e		0.056	0.01	10.18
Bc/Bo	0.203	0.166	0.02	7.98

Source	DF	Sum of Squares	Mean Square	F-Value
Model	1.00	0.46	0.46	63.74
Error	566.00	4.05	0.01	
Total	567.00	4.50		

Dependent Mean	0.09
Root Mean Square Error	0.08
Coefficient of Variation	94.09
R-Square	0.10
Adjusted R-Square	0.10

1800-1861

MULTIPLE LINEAR REGRESSION

Dependent Variable: Rc/Ro

Variable	Mean	Parameter Estimate	Standard Error	T for H0: parameter=0
Intercept e		0.046	0.01	8.03
Bc/Bo	0.154	0.228	0.03	8.15

Source	DF	Sum of Squares	Mean Square	F-Value
Model	1.00	0.32	0.32	66.45
Error	350.00	1.69	0.00	
Total	351.00	2.01		

Dependent Mean	0.08
Root Mean Square Error	0.07
Coefficient of Variation	86.00
R-Square	0.16
Adjusted R-Square	0.16

1861-1865

MULTIPLE LINEAR REGRESSION

Dependent Variable: Rc/Ro

Variable	Mean	Parameter Estimate	Standard Error	T for H0: parameter=0
Intercept e		0.045	0.01	3.55
Bc/Bo	0.132	0.402	0.08	5.21

Source	DF	Sum of Squares	Mean Square	F-Value
Model	1.00	0.08	0.08	27.19
Error	47.00	0.13	0.00	
Total	48.00	0.21		

Dependent Mean	0.10
Root Mean Square Error	0.05
Coefficient of Variation	54.37
R-Square	0.37
Adjusted R-Square	0.35

POST-1865

MULTIPLE LINEAR REGRESSION

Dependent Variable: Rc/Ro

Variable	Mean	Parameter Estimate	Standard Error	T for H0: parameter=0
Intercept e		0.023	0.01	2.51
Bc/Bo	0.125	0.421	0.06	7.40

Source	DF	Sum of Squares	Mean Square	F-Value
Model	1.00	0.22	0.22	54.80
Error	109.00	0.44	0.00	
Total	110.00	0.66		

Dependent Mean	0.08
Root Mean Square Error	0.06
Coefficient of Variation	83.78
R-Square	0.33
Adjusted R-Square	0.33

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1620	47,500	400	32,000	6,000	0.0084	0.1875	2.71	0.39
1621	20,000	5,000	20,000	5,000	0.2500	0.2500	0.00	0.00
1622	26,000	2,000	21,000	8,000	0.0769	0.3810	1.39	0.21
1622	13,000	2,000	17,000	2,000	0.1538	0.1176	0.00	-0.27
1623	28,000	1,000	22,000	4,000	0.0357	0.1818	1.39	0.24
1626	26,000	1,000	11,000	3,000	0.0385	0.2727	1.10	0.86
1626	10,000	800	15,000	2,200	0.0800	0.1467	1.01	-0.41
1628	7,000	1,000	6,000	600	0.1429	0.1000	-0.51	0.15
1630	10,000	500	29,000	800	0.0500	0.0276	0.47	-1.06
1631	36,000	5,000	34,000	8,000	0.1389	0.2353	0.47	0.06
1631	20,000	400	8,000	1,700	0.0200	0.2125	1.45	0.92
1632	60,000	1,000	46,000	2,000	0.0167	0.0435	0.69	0.27
1632	20,000	4,000	25,000	5,000	0.2000	0.2000	0.22	-0.22
1632	33,000	1,000	27,000	3,000	0.0303	0.1111	1.10	0.20
1633	17,000	1,000	15,000	7,000	0.0588	0.4667	1.95	0.13
1634	5,000	100	6,000	1,500	0.0200	0.2500	2.71	-0.18
1634	35,000	2,000	25,000	6,000	0.0571	0.2400	1.10	0.34
1634	30,000	8,000	4,000	1,400	0.2667	0.3500	-1.74	2.01
1634	15,000	1,000	12,000	4,000	0.0667	0.3333	1.39	0.22
1635	34,000	3,000	16,000	5,000	0.0882	0.3125	0.51	0.75
1635	7,000	400	7,000	2,400	0.0571	0.3429	1.79	0.00
1635	4,000	500	6,000	1,500	0.1250	0.2500	1.10	-0.41
1635	5,000	1,000	7,000	1,700	0.2000	0.2429	0.53	-0.34
1635	3,000	600	4,000	1,500	0.2000	0.3750	0.92	-0.29
1636	22,000	7,000	30,000	10,000	0.3182	0.3333	0.36	-0.31
1636	10,000	1,000	12,000	2,000	0.1000	0.1667	0.69	-0.18
1636	5,000	500	3,000	1,000	0.1000	0.3333	0.69	0.51
1637	12,000	3,000	16,000	4,000	0.2500	0.2500	0.29	-0.29
1638	14,000	1,600	17,000	2,000	0.1143	0.1176	0.22	-0.19
1638	5,000	400	4,000	700	0.0800	0.1750	0.56	0.22
1638	5,000	500	7,000	4,000	0.1000	0.5714	2.08	-0.34
1639	14,000	1,500	20,000	6,000	0.1071	0.3000	1.39	-0.36
1639	18,000	600	10,000	1,000	0.0333	0.1000	0.51	0.59
1639	9,000	1,000	18,000	2,000	0.1111	0.1111	0.69	-0.69
1640	10,000	2,000	18,000	4,000	0.2000	0.2222	0.69	-0.59
1641	10,000	1,000	11,000	3,000	0.1000	0.2727	1.10	-0.10
1642	12,000	1,000	25,000	2,000	0.0833	0.0800	0.69	-0.73
1642	20,000	800	18,000	3,000	0.0400	0.1667	1.32	0.11
1642	25,000	5,000	30,000	10,000	0.2000	0.3333	0.69	-0.18
1642	7,500	400	9,000	4,000	0.0533	0.4444	2.30	-0.18

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1642	18,000	1,500	10,000	2,000	0.0833	0.2000	0.29	0.59
1642	12,000	1,000	10,000	4,000	0.0833	0.4000	1.39	0.18
1643	22,000	1,000	18,000	3,000	0.0455	0.1667	1.10	0.20
1643	23,000	4,000	25,000	8,000	0.1739	0.3200	0.69	-0.08
1644	20,000	1,500	18,000	6,000	0.0750	0.3333	1.39	0.11
1644	20,000	8,000	16,000	4,000	0.4000	0.2500	-0.69	0.22
1644	18,000	400	12,000	1,000	0.0222	0.0833	0.92	0.41
1645	15,000	1,000	18,000	3,000	0.0667	0.1667	1.10	-0.18
1645	16,000	2,000	16,000	4,000	0.1250	0.2500	0.69	0.00
1645	18,000	6,000	16,000	4,000	0.3333	0.2500	-0.41	0.12
1645	10,000	700	11,000	1,500	0.0700	0.1364	0.76	-0.10
1646	17,000	1,000	12,000	4,000	0.0588	0.3333	1.39	0.35
1647	20,000	2,000	10,000	1,800	0.1000	0.1800	-0.11	0.69
1647	4,000	2,000	16,000	8,000	0.5000	0.5000	1.39	-1.39
1648	14,000	4,000	18,000	5,000	0.2857	0.2778	0.22	-0.25
1648	12,000	1,000	11,000	3,000	0.0833	0.2727	1.10	0.09
1650	11,000	1,000	23,000	3,000	0.0909	0.1304	1.10	-0.74
1650	11,000	1,000	9,000	2,000	0.0909	0.2222	0.69	0.20
1651	28,000	1,000	16,000	3,000	0.0357	0.1875	1.10	0.56
1652	12,000	4,000	6,000	2,000	0.3333	0.3333	-0.69	0.69
1653	15,000	1,000	25,000	2,000	0.0667	0.0800	0.69	-0.51
1656	22,000	1,500	25,000	2,000	0.0682	0.0800	0.29	-0.13
1657	2,500	200	10,000	3,000	0.0800	0.3000	2.71	-1.39
1658	15,000	2,000	12,000	4,000	0.1333	0.3333	0.69	0.22
1659	9,000	2,000	6,000	2,000	0.2222	0.3333	0.00	0.41
1664	30,000	2,000	60,000	8,666	0.0667	0.1444	1.47	-0.69
1665	14,000	1,000	18,000	4,000	0.0714	0.2222	1.39	-0.25
1667	130,000	108,000	20,000	16,000	0.8308	0.8000	-1.91	1.87
1673	50,000	1,000	80,000	20,000	0.0200	0.2500	3.00	-0.47
1674	4,000	300	5,000	300	0.0750	0.0600	0.00	-0.22
1674	50,000	6,000	70,000	8,600	0.1200	0.1229	0.36	-0.34
1675	22,000	3,000	14,000	4,000	0.1364	0.2857	0.29	0.45
1675	6,000	500	6,400	2,000	0.0833	0.3125	1.39	-0.06
1675	17,000	1,100	14,000	2,500	0.0647	0.1786	0.82	0.19
1676	16,000	4,000	12,000	4,000	0.2500	0.3333	0.00	0.29
1677	8,000	1,000	7,000	2,000	0.1250	0.2857	0.69	0.13
1677	8,000	1,300	12,000	3,000	0.1625	0.2500	0.84	-0.41
1677	30,000	4,400	30,000	7,000	0.1467	0.2333	0.46	0.00
1678	40,000	4,000	40,000	4,000	0.1000	0.1000	0.00	0.00
1683	10,000	5,000	200,000	48,000	0.5000	0.2400	2.26	-3.00

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1683	28,000	1,000	16,000	9,000	0.0357	0.5625	2.20	0.56
1683	76,000	5,000	107,000	15,000	0.0658	0.1402	1.10	-0.34
1684	32,000	100	18,000	3,000	0.0031	0.1667	3.40	0.58
1685	60,000	100	42,000	2,000	0.0017	0.0476	3.00	0.36
1686	50,000	500	80,000	2,000	0.0100	0.0250	1.39	-0.47
1687	50,000	2,000	60,000	8,000	0.0400	0.1333	1.39	-0.18
1688	3,000	200	15,000	5,000	0.0667	0.3333	3.22	-1.61
1688	53,000	1,300	8,300	7,000	0.0245	0.8434	1.68	1.85
1689	17,000	400	40,000	10,000	0.0235	0.2500	3.22	-0.86
1689	20,000	100	5,000	2,000	0.0050	0.4000	3.00	1.39
1689	18,000	400	32,000	3,000	0.0222	0.0938	2.01	-0.58
1690	35,000	500	23,000	1,500	0.0143	0.0652	1.10	0.42
1690	50,000	9,500	5,000	4,500	0.1900	0.9000	-0.75	2.30
1690	12,000	2,000	18,000	2,800	0.1667	0.1556	0.34	-0.41
1690	50,000	3,000	38,000	6,000	0.0600	0.1579	0.69	0.27
1691	20,000	2,700	25,000	4,400	0.1350	0.1760	0.49	-0.22
1691	4,000	700	12,000	1,500	0.1750	0.1250	0.76	-1.10
1691	50,000	8,000	100,000	20,000	0.1600	0.2000	0.92	-0.69
1692	46,000	7,000	8,300	4,000	0.1522	0.4819	-0.56	1.71
1692	57,000	7,000	63,000	6,600	0.1228	0.1048	-0.06	-0.10
1693	80,000	8,000	50,000	12,000	0.1000	0.2400	0.41	0.47
1693	40,000	3,000	36,000	9,000	0.0750	0.2500	1.10	0.11
1694	26,000	1,300	20,000	3,500	0.0500	0.1750	0.99	0.26
1694	80,000	18,000	13,000	8,0[illegible]	0.2250	0.6154	-0.81	1.82
1696	60,000	4,000	50,000	4,[illegible]	0.0667	0.0800	0.00	0.18
1697	50,000	2,100	100,000	30,000	0.0420	0.3000	2.66	-0.69
1697	30,000	10,000	12,000	7,000	0.3333	0.5833	-0.36	0.92
1700	9,000	2,000	40,000	8,000	0.2222	0.2000	1.39	-1.49
1701	6,000	800	8,000	1,000	0.1333	0.1250	0.22	-0.29
1701	8,000	1,200	20,000	2,800	0.1500	0.1400	0.85	-0.92
1701	15,000	100	5,000	350	0.0067	0.0700	1.25	1.10
1701	22,000	200	38,000	3,600	0.0091	0.0947	2.89	-0.55
1702	46,000	3,000	4,400	1,700	0.0652	0.3864	-0.57	2.35
1702	25,000	2,700	30,000	3,500	0.1080	0.1167	0.26	-0.18
1702	17,000	2,700	14,000	1,850	0.1588	0.1321	-0.38	0.19
1702	4,000	200	2,800	600	0.0500	0.2143	1.10	0.36
1702	15,000	1,400	4,000	1,200	0.0933	0.3000	-0.15	1.32
1702	12,000	1,200	5,000	1,400	0.1000	0.2800	0.15	0.88
1702	10,000	1,200	8,000	800	0.1200	0.1000	-0.41	0.22
1702	25,000	200	23,000	400	0.0080	0.0174	0.69	0.08

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1702	12,000	1,100	22,000	2,000	0.0917	0.0909	0.60	-0.61
1703	19,000	2,300	15,000	2,500	0.1211	0.1667	0.08	0.24
1703	26,000	5,000	5,600	1,800	0.1923	0.3214	-1.02	1.54
1703	18,000	4,000	22,000	4,000	0.2222	0.1818	0.00	-0.20
1703	24,000	900	3,500	300	0.0375	0.0857	-1.10	1.93
1703	12,000	500	10,000	1,200	0.0417	0.1200	0.88	0.18
1703	2,000	100	12,000	1,300	0.0500	0.1083	2.56	-1.79
1703	4,200	500	5,000	800	0.1190	0.1600	0.47	-0.17
1703	1,300	200	8,700	2,400	0.1538	0.2759	2.48	-1.90
1703	7,000	1,300	2,500	700	0.1857	0.2800	-0.62	1.03
1703	14,000	4,000	6,000	1,000	0.2857	0.1667	-1.39	0.85
1703	24,000	200	35,000	3,000	0.0083	0.0857	2.71	-0.38
1703	23,000	1,500	18,000	4,500	0.0652	0.2500	1.10	0.25
1704	25,000	6,000	11,000	2,000	0.2400	0.1818	-1.10	0.82
1704	7,000	500	20,000	2,000	0.0714	0.1000	1.39	-1.05
1704	3,000	100	12,000	4,000	0.0333	0.3333	3.69	-1.39
1704	10,000	400	2,400	1,600	0.0400	0.6667	1.39	1.43
1704	3,600	200	7,400	800	0.0556	0.1081	1.39	-0.72
1704	6,500	600	9,000	2,000	0.0923	0.2222	1.20	-0.33
1704	25,000	3,000	4,500	2,500	0.1200	0.5556	-0.18	1.71
1704	10,000	1,500	4,000	600	0.1500	0.1500	-0.92	0.92
1704	50,000	12,500	52,000	14,000	0.2500	0.2692	0.11	-0.04
1704	26,000	1,100	7,000	900	0.0423	0.1286	-0.20	1.31
1705	14,000	200	15,000	2,000	0.0143	0.1333	2.30	-0.07
1705	2,000	50	4,000	3,500	0.0250	0.8750	4.25	-0.69
1705	7,000	1,000	20,000	5,000	0.1429	0.2500	1.61	-1.05
1705	13,000	600	24,000	6,000	0.0462	0.2500	2.30	-0.61
1705	22,000	3,000	24,000	4,000	0.1364	0.1667	0.29	-0.09
1706	36,000	1,000	12,000	1,400	0.0278	0.1167	0.34	1.10
1706	30,000	3,000	5,500	1,000	0.1000	0.1818	-1.10	1.70
1706	30,000	4,300	42,000	3,800	0.1433	0.0905	-0.12	-0.34
1706	10,500	3,000	36,000	14,000	0.2857	0.3889	1.54	-1.23
1706	20,000	500	5,000	300	0.0250	0.0600	-0.51	1.39
1706	5,000	200	2,000	600	0.0400	0.3000	1.10	0.92
1706	15,000	1,000	10,000	1,500	0.0667	0.1500	0.41	0.41
1706	12,000	2,000	18,000	5,000	0.1667	0.2778	0.92	-0.41
1706	60,000	5,000	62,000	2,000	0.0833	0.0323	-0.92	-0.03
1706	41,000	500	19,000	3,000	0.0122	0.1579	1.79	0.77
1707	21,000	2,000	16,000	5,000	0.0952	0.3125	0.92	0.27
1707	3,500	750	3,000	750	0.2143	0.2500	0.00	0.15

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1708	7,000	1,600	33,000	900	0.2286	0.0273	-0.58	-1.55
1708	35,000	14,000	16,000	7,000	0.4000	0.4375	-0.69	0.78
1708	7,000	300	15,000	2,000	0.0429	0.1333	1.90	-0.76
1708	10,000	500	16,000	6,000	0.0500	0.3750	2.48	-0.47
1708	3,000	200	1,200	200	0.0667	0.1667	0.00	0.92
1708	12,000	2,000	6,000	3,000	0.1667	0.5000	0.41	0.69
1708	7,000	1,200	3,500	3,000	0.1714	0.8571	0.92	0.69
1708	17,000	4,000	13,000	5,000	0.2353	0.3846	0.22	0.27
1708	90,000	6,000	80,000	6,000	0.0667	0.0750	0.00	0.12
1708	11,000	1,000	22,000	2,500	0.0909	0.1136	0.92	-0.69
1709	40,000	5,400	7,000	3,200	0.1350	0.4571	-0.52	1.74
1709	6,000	400	7,000	2,600	0.0667	0.3714	1.87	-0.15
1709	93,000	25,000	90,000	11,000	0.2688	0.1222	-0.82	0.03
1709	16,000	400	20,000	17,000	0.0250	0.8500	3.75	-0.22
1709	54,000	5,000	26,000	9,200	0.0926	0.3538	0.61	0.73
1710	60,000	8,000	8,000	3,000	0.1333	0.3750	-0.98	2.01
1710	21,000	4,000	13,600	3,000	0.1905	0.2206	-0.29	0.43
1710	28,000	7,000	7,000	3,400	0.2500	0.4857	-0.72	1.39
1710	9,000	900	3,000	1,000	0.1000	0.3333	0.11	1.10
1710	12,000	1,500	4,000	600	0.1250	0.1500	-0.92	1.10
1710	24,000	400	22,000	1,500	0.0167	0.0682	1.32	0.09
1710	22,000	1,600	20,000	5,000	0.0727	0.2500	1.14	0.10
1710	31,000	3,000	4,000	1,800	0.0968	0.4500	-0.51	2.05
1710	14,000	3,000	11,000	4,000	0.2143	0.3636	0.29	0.24
1711	260,000	7,000	40,000	2,100	0.0269	0.0525	-1.20	1.87
1711	30,000	3,000	5,000	1,800	0.1000	0.3600	-0.51	1.79
1711	2,500	800	8,000	2,500	0.3200	0.3125	1.14	-1.16
1712	22,000	400	7,000	200	0.0182	0.0286	-0.69	1.15
1712	25,000	500	3,200	1,000	0.0200	0.3125	0.69	2.06
1712	14,000	1,700	12,000	4,000	0.1214	0.3333	0.86	0.15
1712	40,000	2,200	18,000	3,000	0.0550	0.1667	0.31	0.80
1712	30,000	2,100	18,000	2,300	0.0700	0.1278	0.09	0.51
1712	18,000	3,000	5,500	2,000	0.1667	0.3636	-0.41	1.19
1712	28,000	1,000	2,200	700	0.0357	0.3182	-0.36	2.54
1713	40,000	10,000	7,000	2,000	0.2500	0.2857	-1.61	1.74
1713	80,000	10,000	9,300	3,600	0.1250	0.3871	-1.02	2.15
1714	40,000	20,000	16,000	6,000	0.5000	0.3750	-1.20	0.92
1715	18,000	500	5,000	2,500	0.0278	0.5000	1.61	1.28
1715	22,000	1,200	7,000	2,000	0.0545	0.2857	0.51	1.15
1715	10,000	1,000	3,000	700	0.1000	0.2333	-0.36	1.20

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1715	70,000	10,000	12,000	6,000	0.1429	0.5000	-0.51	1.76
1716	45,000	4,500	18,000	3,000	0.1000	0.1667	-0.41	0.92
1716	63,000	4,500	60,000	6,000	0.0714	0.1000	0.29	0.05
1717	23,000	2,000	1,000	600	0.0870	0.6000	-1.20	3.14
1717	100,000	20,000	30,000	5,000	0.2000	0.1667	-1.39	1.20
1717	50,000	5,400	150,000	15,000	0.1080	0.1000	1.02	-1.10
1718	9,300	1,500	6,000	1,500	0.1613	0.2500	0.00	0.44
1719	29,000	2,000	21,000	800	0.0690	0.0381	-0.92	0.32
1719	18,000	5,200	4,000	900	0.2889	0.2250	-1.75	1.50
1724	16,600	800	6,200	1,000	0.0482	0.1613	0.22	0.98
1734	10,000	1,500	6,000	1,000	0.1500	0.1667	-0.41	0.51
1734	53,000	4,000	37,000	6,000	0.0755	0.1622	0.41	0.36
1734	20,000	800	40,000	1,100	0.0400	0.0275	0.32	-0.69
1734	117,000	10,000	4,500	1,000	0.0855	0.2222	-2.30	3.26
1734	40,000	5,900	27,000	5,800	0.1475	0.2148	-0.02	0.39
1735	6,000	2,500	3,000	1,400	0.4167	0.4667	-0.58	0.69
1737	80,000	4,000	22,000	17,000	0.0500	0.7727	1.45	1.29
1737	6,000	2,000	20,000	10,000	0.3333	0.5000	1.61	-1.20
1738	10,000	1,000	20,000	5,000	0.1000	0.2500	1.61	-0.69
1738	40,000	1,300	17,000	2,000	0.0325	0.1176	0.43	0.86
1739	80,000	8,000	40,000	5,200	0.1000	0.1300	-0.43	0.69
1739	60,000	100	100,000	2,000	0.0017	0.0200	3.00	-0.51
1741	21,600	3,900	15,800	3,000	0.1806	0.1899	-0.26	0.31
1741	10,000	3,000	5,000	2,500	0.3000	0.5000	-0.18	0.69
1741	5,000	50	1,200	60	0.0100	0.0500	0.18	1.43
1742	4,000	100	7,000	150	0.0250	0.0214	0.41	-0.56
1742	18,000	550	8,500	250	0.0306	0.0294	-0.79	0.75
1742	28,000	4,200	28,000	3,000	0.1500	0.1071	-0.34	0.00
1743	35,000	2,050	26,000	2,800	0.0586	0.1077	0.31	0.30
1743	5,400	100	6,000	600	0.0185	0.1000	1.79	-0.11
1743	13,000	3,200	11,000	1,600	0.2462	0.1455	-0.69	0.17
1743	32,000	100	9,000	1,000	0.0031	0.1111	2.30	1.27
1744	80,000	150	14,900	1,400	0.0019	0.0940	2.23	1.68
1744	24,000	3,000	6,000	1,100	0.1250	0.1833	-1.00	1.39
1744	40,000	1,500	10,000	650	0.0375	0.0650	-0.84	1.39
1744	24,000	1,400	16,000	700	0.0583	0.0438	-0.69	0.41
1744	70,000	16,000	7,000	2,200	0.2286	0.3143	-1.98	2.30
1744	26,000	4,000	25,000	3,600	0.1538	0.1440	-0.11	0.04
1745	70,000	4,800	75,000	9,600	0.0686	0.1280	0.69	-0.07
1745	2,500	100	2,300	400	0.0400	0.1739	1.39	0.08

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1745	60,000	5,600	50,000	9,000	0.0933	0.1800	0.47	0.18
1745	17,000	800	7,000	2,400	0.0471	0.3429	1.10	0.89
1745	50,000	1,000	30,000	1,000	0.0200	0.0333	0.00	0.51
1745	50,000	2,000	4,000	600	0.0400	0.1500	-1.20	2.53
1745	35,000	5,100	35,000	3,800	0.1457	0.1086	-0.29	0.00
1745	22,000	3,700	38,000	4,500	0.1682	0.1184	0.20	-0.55
1745	8,500	120	5,500	500	0.0141	0.0909	1.43	0.44
1746	10,000	300	6,000	1,000	0.0300	0.1667	1.20	0.51
1746	9,000	400	4,000	1,600	0.0444	0.4000	1.39	0.81
1746	40,000	3,000	44,000	7,000	0.0750	0.1591	0.85	-0.10
1746	60,000	1,000	12,000	500	0.0167	0.0417	-0.69	1.61
1746	25,000	800	41,000	300	0.0320	0.0073	-0.98	-0.49
1746	30,000	2,500	25,000	4,500	0.0833	0.1800	0.59	0.18
1746	110,000	4,000	75,000	7,000	0.0364	0.0933	0.56	0.38
1747	98,000	10,000	82,000	9,000	0.1020	0.1098	-0.11	0.18
1747	35,000	6,500	26,000	5,500	0.1857	0.2115	-0.17	0.30
1747	7,000	300	14,000	5,700	0.0429	0.4071	2.94	-0.69
1748	80,000	2,000	13,000	2,000	0.0250	0.1538	0.00	1.82
1749	20,000	700	9,000	400	0.0350	0.0444	-0.56	0.80
1753	11,000	600	7,000	900	0.0545	0.1286	0.41	0.45
1755	1,000	100	2,300	1,500	0.1000	0.6522	2.71	-0.83
1756	12,000	2,000	2,800	400	0.1667	0.1429	-1.61	1.46
1756	30,000	2,600	33,000	2,200	0.0867	0.0667	-0.17	-0.10
1757	32,000	1,500	14,000	1,500	0.0469	0.1071	0.00	0.83
1757	55,000	6,000	25,000	4,100	0.1091	0.1640	-0.38	0.79
1757	15,000	700	14,000	400	0.0467	0.0286	-0.56	0.07
1757	80,000	5,300	30,000	9,666	0.0663	0.3222	0.60	0.98
1757	5,000	1,000	3,000	1,500	0.2000	0.5000	0.41	0.51
1757	50,000	1,100	63,000	3,000	0.0220	0.0476	1.00	-0.23
1757	60,000	2,350	36,000	1,250	0.0392	0.0347	-0.63	0.51
1757	64,000	12,600	61,000	9,200	0.1969	0.1508	-0.31	0.05
1757	54,000	6,400	33,000	8,600	0.1185	0.2606	0.30	0.49
1757	22,000	600	41,000	3,400	0.0273	0.0829	1.73	-0.62
1757	35,000	6,200	65,000	10,000	0.1771	0.1538	0.48	-0.62
1758	10,000	300	8,000	800	0.0300	0.1000	0.98	0.22
1758	3,000	200	6,000	400	0.0667	0.0667	0.69	-0.69
1758	3,500	100	2,800	750	0.0286	0.2679	2.01	0.22
1758	11,000	600	6,700	1,100	0.0545	0.1642	0.61	0.50
1758	2,600	300	6,500	400	0.1154	0.0615	0.29	-0.92
1758	3,600	400	15,000	2,000	0.1111	0.1333	1.61	-1.43

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1758	7,500	2,200	45,000	800	0.2933	0.0178	-1.01	-1.79
1758	30,000	600	18,000	700	0.0200	0.0389	0.15	0.51
1758	33,000	11,000	52,000	18,000	0.3333	0.3462	0.49	-0.45
1758	65,000	5,400	42,000	7,900	0.0831	0.1881	0.38	0.44
1758	31,000	1,800	47,000	5,200	0.0581	0.1106	1.06	-0.42
1758	4,000	1,000	7,300	600	0.2500	0.0822	-0.51	-0.60
1759	38,000	2,700	52,000	4,900	0.0711	0.0942	0.60	-0.31
1759	12,000	50	9,000	100	0.0042	0.0111	0.69	0.29
1759	12,000	200	4,000	300	0.0167	0.0750	0.41	1.10
1759	5,000	200	10,000	350	0.0400	0.0350	0.56	-0.69
1759	70,000	16,000	48,000	18,700	0.2286	0.3896	0.16	0.38
1759	8,000	650	4,500	1,500	0.0813	0.3333	0.84	0.58
1759	72,000	4,800	28,000	7,000	0.0667	0.2500	0.38	0.94
1759	38,000	1,000	13,500	1,300	0.0263	0.0963	0.26	1.03
1759	26,000	900	13,500	500	0.0346	0.0370	-0.59	0.66
1759	36,000	1,800	27,000	2,500	0.0500	0.0926	0.33	0.29
1759	30,000	200	4,000	1,800	0.0067	0.4500	2.20	2.01
1759	20,000	2,700	16,000	1,300	0.1350	0.0813	-0.73	0.22
1760	3,000	300	5,000	300	0.1000	0.0600	0.00	-0.51
1760	13,000	200	8,000	1,200	0.0154	0.1500	1.79	0.49
1760	44,000	10,000	66,000	9,000	0.2273	0.1364	-0.11	-0.41
1760	7,000	800	6,000	1,200	0.1143	0.2000	0.41	0.15
1760	30,000	800	16,000	650	0.0267	0.0406	-0.21	0.63
1760	19,000	1,300	17,000	1,500	0.0684	0.0882	0.14	0.11
1760	22,000	2,000	12,000	1,000	0.0909	0.0833	-0.69	0.61
1760	38,700	3,000	10,800	6,000	0.0775	0.5556	0.69	1.28
1760	30,000	3,300	30,000	3,800	0.1100	0.1267	0.14	0.00
1760	5,400	200	2,800	500	0.0370	0.1786	0.92	0.66
1761	10,000	300	4,000	600	0.0300	0.1500	0.69	0.92
1761	12,500	400	5,500	600	0.0320	0.1091	0.41	0.82
1761	12,000	400	15,000	2,500	0.0333	0.1667	1.83	-0.22
1761	14,000	1,000	7,000	800	0.0714	0.1143	-0.22	0.69
1761	14,000	1,700	4,000	400	0.1214	0.1000	-1.45	1.25
1761	52,000	1,300	100,000	3,000	0.0250	0.0300	0.84	-0.65
1762	12,000	1,100	10,000	700	0.0917	0.0700	-0.45	0.18
1762	24,000	700	16,000	500	0.0292	0.0313	-0.34	0.41
1762	7,000	300	5,000	1,400	0.0429	0.2800	1.54	0.34
1762	17,000	750	6,000	700	0.0441	0.1167	-0.07	1.04
1762	50,000	1,000	30,000	1,600	0.0200	0.0533	0.47	0.51
1762	8,000	1,500	10,000	600	0.1875	0.0600	-0.92	-0.22

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1762	17,000	3,000	12,500	3,500	0.1765	0.2800	0.15	0.31
1762	27,000	1,600	31,000	3,000	0.0593	0.0968	0.63	-0.14
1762	16,000	1,100	32,000	1,200	0.0688	0.0375	0.09	-0.69
1762	23,000	1,600	20,000	1,750	0.0696	0.0875	0.09	0.14
1762	57,000	300	25,000	1,500	0.0053	0.0600	1.61	0.82
1762	17,000	100	3,000	200	0.0059	0.0667	0.69	1.73
1762	16,000	450	13,000	800	0.0281	0.0615	0.58	0.21
1769	30,000	600	80,000	3,000	0.0200	0.0375	1.61	-0.98
1770	10,000	400	16,000	3,000	0.0400	0.1875	2.01	-0.47
1770	40,000	1,000	15,000	20,000	0.0250	1.3333	3.00	0.98
1770	12,000	2,500	12,000	6,000	0.2083	0.5000	0.88	0.00
1771	10,000	1,000	10,000	4,000	0.1000	0.4000	1.39	0.00
1771	15,000	500	40,000	2,000	0.0333	0.0500	1.39	-0.98
1771	32,000	2,000	57,000	4,000	0.0625	0.0702	0.69	-0.58
1773	6,000	800	10,000	2,000	0.1333	0.2000	0.92	-0.51
1775	2,000	1,150	1,200	450	0.5750	0.3750	-0.94	0.51
1776	6,000	430	2,000	200	0.0717	0.1000	-0.77	1.10
1776	25,000	320	11,000	2,000	0.0128	0.1818	1.83	0.82
1776	2,400	10	1,500	1,300	0.0042	0.8667	4.87	0.47
1777	15,000	500	10,000	1,200	0.0333	0.1200	0.88	0.41
1777	5,000	600	6,000	600	0.1200	0.1000	0.00	-0.18
1777	18,000	600	8,000	900	0.0333	0.1125	0.41	0.81
1779	3,200	20	10,000	800	0.0063	0.0800	3.69	-1.14
1779	11,000	300	3,000	400	0.0273	0.1333	0.29	1.30
1780	2,000	300	4,000	900	0.1500	0.2250	1.10	-0.69
1780	10,000	300	7,000	1,000	0.0300	0.1429	1.20	0.36
1781	2,400	600	5,000	700	0.2500	0.1400	0.15	-0.73
1782	20,000	1,100	10,000	700	0.0550	0.0700	-0.45	0.69
1782	7,000	1,300	31,000	6,000	0.1857	0.1935	1.53	-1.49
1787	12,000	1,000	5,000	3,000	0.0833	0.6000	1.10	0.88
1788	8,000	1,600	8,000	550	0.2000	0.0688	-1.07	0.00
1788	90,000	4,800	14,000	10,000	0.0533	0.7143	0.73	1.86
1789	23,000	300	30,000	1,000	0.0130	0.0333	1.20	-0.27
1789	27,000	600	50,000	5,000	0.0222	0.1000	2.12	-0.62
1790	7,000	100	7,000	2,000	0.0143	0.2857	3.00	0.00
1790	32,000	10,000	40,000	31,000	0.3125	0.7750	1.13	-0.22
1791	36,000	1,000	64,000	3,000	0.0278	0.0469	1.10	-0.58
1791	18,000	4,000	15,000	8,000	0.2222	0.5333	0.69	0.18
1791	18,000	500	15,000	1,500	0.0278	0.1000	1.10	0.18
1792	45,000	2,000	13,200	1,000	0.0444	0.0758	-0.69	1.23

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1792	52,000	300	35,000	200	0.0058	0.0057	-0.41	0.40
1792	13,000	200	1,500	180	0.0154	0.1200	-0.11	2.16
1792	6,000	1,000	10,000	1,000	0.1667	0.1000	0.00	-0.51
1793	10,000	500	35,000	5,000	0.0500	0.1429	2.30	-1.25
1793	30,000	1,500	13,000	3,100	0.0500	0.2385	0.73	0.84
1793	32,000	2,000	18,000	4,000	0.0625	0.2222	0.69	0.58
1793	31,000	2,100	25,000	4,000	0.0677	0.1600	0.64	0.22
1793	43,000	3,000	22,000	4,000	0.0698	0.1818	0.29	0.67
1793	20,000	2,000	20,000	5,000	0.1000	0.2500	0.92	0.00
1793	20,000	2,000	25,000	8,000	0.1000	0.3200	1.39	-0.22
1793	45,000	5,000	30,000	2,500	0.1111	0.0833	-0.69	0.41
1793	24,000	3,000	16,000	1,600	0.1250	0.1000	-0.63	0.41
1793	53,000	1,000	27,000	3,000	0.0189	0.1111	1.10	0.67
1793	25,000	4,000	40,000	8,000	0.1600	0.2000	0.69	-0.47
1793	12,000	2,000	38,000	5,000	0.1667	0.1316	0.92	-1.15
1793	17,000	2,000	22,000	4,000	0.1176	0.1818	0.69	-0.26
1793	38,000	900	22,000	2,000	0.0237	0.0909	0.80	0.55
1793	5,000	200	25,000	1,500	0.0400	0.0600	2.01	-1.61
1793	12,000	800	15,000	200	0.0667	0.0133	-1.39	-0.22
1793	10,500	900	6,000	600	0.0857	0.1000	-0.41	0.56
1793	10,000	100	1,200	300	0.0100	0.2500	1.10	2.12
1793	8,000	100	10,000	2,500	0.0125	0.2500	3.22	-0.22
1793	8,000	200	34,000	1,200	0.0250	0.0353	1.79	-1.45
1793	8,000	200	6,000	1,200	0.0250	0.2000	1.79	0.29
1793	8,000	200	4,000	1,200	0.0250	0.3000	1.79	0.69
1793	20,000	500	8,000	4,000	0.0250	0.5000	2.08	0.92
1793	20,000	500	7,000	6,000	0.0250	0.8571	2.48	1.05
1793	25,000	1,000	15,000	1,500	0.0400	0.1000	0.41	0.51
1793	11,000	300	6,000	3,000	0.0273	0.5000	2.30	0.61
1793	35,000	500	30,000	1,500	0.0143	0.0500	1.10	0.15
1793	8,000	300	5,000	4,000	0.0375	0.8000	2.59	0.47
1793	2,500	100	8,000	2,000	0.0400	0.2500	3.00	-1.16
1793	10,000	500	6,000	800	0.0500	0.1333	0.47	0.51
1793	15,000	300	16,000	1,200	0.0200	0.0750	1.39	-0.06
1793	7,000	400	18,000	3,000	0.0571	0.1667	2.01	-0.94
1793	8,000	500	6,000	1,000	0.0625	0.1667	0.69	0.29
1793	12,000	800	7,000	800	0.0667	0.1143	0.00	0.54
1793	8,000	600	16,000	2,000	0.0750	0.1250	1.20	-0.69
1793	12,000	1,000	14,000	2,000	0.0833	0.1429	0.69	-0.15
1793	60,000	600	30,000	1,500	0.0100	0.0500	0.92	0.69

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1793	9,000	1,100	11,000	2,000	0.1222	0.1818	0.60	-0.20
1793	26,000	800	32,000	24,000	0.0308	0.7500	3.40	-0.21
1793	20,000	1,200	15,000	700	0.0600	0.0467	-0.54	0.29
1793	36,000	50	9,000	2,000	0.0014	0.2222	3.69	1.39
1793	43,000	2,600	41,000	3,000	0.0605	0.0732	0.14	0.05
1793	43,000	1,800	50,000	2,000	0.0419	0.0400	0.11	-0.15
1793	35,000	1,000	14,000	4,000	0.0286	0.2857	1.39	0.92
1793	24,000	1,300	8,000	1,000	0.0542	0.1250	-0.26	1.10
1794	28,000	1,500	13,000	2,000	0.0536	0.1538	0.29	0.77
1794	26,000	1,500	26,000	3,000	0.0577	0.1154	0.69	0.00
1794	28,000	400	60,000	3,000	0.0143	0.0500	2.01	-0.76
1794	41,000	3,000	73,000	2,000	0.0732	0.0274	-0.41	-0.58
1794	22,500	2,800	44,000	4,000	0.1244	0.0909	0.36	-0.67
1794	35,000	3,000	50,000	10,000	0.0857	0.2000	1.20	-0.36
1794	22,000	4,000	28,000	8,000	0.1818	0.2857	0.69	-0.24
1794	88,000	1,500	77,000	3,000	0.0170	0.0390	0.69	0.13
1794	9,000	50	6,000	600	0.0056	0.1000	2.48	0.41
1794	20,000	500	30,000	900	0.0250	0.0300	0.59	-0.41
1794	7,000	300	15,000	1,200	0.0429	0.0800	1.39	-0.76
1794	18,000	1,000	11,000	800	0.0556	0.0727	-0.22	0.49
1794	81,000	5,000	46,000	5,000	0.0617	0.1087	0.00	0.57
1794	10,000	800	20,000	1,400	0.0800	0.0700	0.56	-0.69
1794	20,000	1,000	20,000	600	0.0500	0.0300	-0.51	0.00
1794	20,000	1,000	20,000	2,000	0.0500	0.1000	0.69	0.00
1794	14,000	500	2,000	500	0.0357	0.2500	0.00	1.95
1794	12,000	500	10,000	3,000	0.0417	0.3000	1.79	0.18
1794	12,000	600	12,000	1,300	0.0500	0.1083	0.77	0.00
1794	2,000	100	7,000	1,300	0.0500	0.1857	2.56	-1.25
1794	30,000	400	16,000	1,000	0.0133	0.0625	0.92	0.63
1794	60,000	1,000	60,000	2,500	0.0167	0.0417	0.92	0.00
1794	20,000	1,500	8,000	3,000	0.0750	0.3750	0.69	0.92
1794	12,000	1,000	10,000	6,000	0.0833	0.6000	1.79	0.18
1794	10,000	1,000	7,000	2,000	0.1000	0.2857	0.69	0.36
1794	28,000	1,000	42,000	2,000	0.0357	0.0476	0.69	-0.41
1794	35,000	1,000	18,000	1,500	0.0286	0.0833	0.41	0.66
1794	9,000	1,000	6,000	1,500	0.1111	0.2500	0.41	0.41
1794	70,000	3,000	74,000	4,000	0.0429	0.0541	0.29	-0.06
1794	90,000	1,500	90,000	7,000	0.0167	0.0778	1.54	0.00
1794	8,000	1,000	12,000	4,000	0.1250	0.3333	1.39	-0.41
1794	6,000	1,000	5,000	2,000	0.1667	0.4000	0.69	0.18

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1794	47,000	500	7,000	2,000	0.0106	0.2857	1.39	1.90
1794	60,000	1,000	13,000	4,000	0.0167	0.3077	1.39	1.53
1794	50,000	600	5,000	1,000	0.0120	0.2000	0.51	2.30
1794	24,000	1,300	19,000	900	0.0542	0.0474	-0.37	0.23
1794	25,000	1,000	10,000	1,500	0.0400	0.1500	0.41	0.92
1794	50,000	3,000	45,000	5,500	0.0600	0.1222	0.61	0.11
1794	30,000	1,500	10,000	1,500	0.0500	0.1500	0.00	1.10
1795	35,000	600	25,000	2,500	0.0171	0.1000	1.43	0.34
1795	42,000	300	60,000	700	0.0071	0.0117	0.85	-0.36
1795	25,000	2,500	18,000	3,000	0.1000	0.1667	0.18	0.33
1795	43,000	600	37,000	1,100	0.0140	0.0297	0.61	0.15
1795	6,000	200	12,000	1,000	0.0333	0.0833	1.61	-0.69
1795	13,000	500	17,000	1,700	0.0385	0.1000	1.22	-0.27
1795	27,000	650	12,000	1,500	0.0241	0.1250	0.84	0.81
1795	36,000	1,400	33,000	3,000	0.0389	0.0909	0.76	0.09
1796	50,000	1,200	48,000	1,100	0.0240	0.0229	-0.09	0.04
1796	28,000	5,600	21,000	3,000	0.2000	0.1429	-0.62	0.29
1796	25,000	600	9,000	550	0.0240	0.0611	-0.09	1.02
1796	25,000	1,000	12,000	900	0.0400	0.0750	-0.11	0.73
1796	15,000	300	14,000	400	0.0200	0.0286	0.29	0.07
1796	20,000	750	10,000	1,000	0.0375	0.1000	0.29	0.69
1796	17,500	900	9,500	400	0.0514	0.0421	-0.81	0.61
1796	4,600	400	34,000	12,000	0.0870	0.3529	3.40	-2.00
1796	15,000	300	6,000	1,100	0.0200	0.1833	1.30	0.92
1796	35,000	100	25,000	2,000	0.0029	0.0800	3.00	0.34
1796	36,000	400	11,000	500	0.0111	0.0455	0.22	1.19
1796	24,000	600	12,000	1,600	0.0250	0.1333	0.98	0.69
1796	28,000	1,000	32,000	1,000	0.0357	0.0313	0.00	-0.13
1796	26,000	650	24,000	1,000	0.0250	0.0417	0.43	0.08
1796	27,000	500	10,300	300	0.0185	0.0291	-0.51	0.96
1796	14,000	800	4,000	2,500	0.0571	0.6250	1.14	1.25
1796	9,000	700	4,000	3,000	0.0778	0.7500	1.46	0.81
1796	44,000	1,200	30,000	2,000	0.0273	0.0667	0.51	0.38
1796	15,000	1,500	8,000	3,500	0.1000	0.4375	0.85	0.63
1796	35,000	500	23,000	300	0.0143	0.0130	-0.51	0.42
1796	30,000	600	24,000	2,300	0.0200	0.0958	1.34	0.22
1796	50,000	2,000	11,000	400	0.0400	0.0364	-1.61	1.51
1796	14,000	2,200	8,000	800	0.1571	0.1000	-1.01	0.56
1796	19,500	2,100	10,500	4,400	0.1077	0.4190	0.74	0.62
1796	24,000	500	14,000	1,000	0.0208	0.0714	0.69	0.54

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1796	36,000	2,000	45,000	1,300	0.0556	0.0289	-0.43	-0.22
1796	22,000	800	10,000	1,200	0.0364	0.1200	0.41	0.79
1796	7,000	1,200	5,000	2,000	0.1714	0.4000	0.51	0.34
1796	36,000	800	34,000	1,200	0.0222	0.0353	0.41	0.06
1796	20,000	400	16,000	600	0.0200	0.0375	0.41	0.22
1796	20,000	1,500	23,000	3,000	0.0750	0.1304	0.69	-0.14
1796	20,000	1,500	14,000	2,500	0.0750	0.1786	0.51	0.36
1796	40,000	400	24,000	600	0.0100	0.0250	0.41	0.51
1796	20,000	2,000	15,000	3,000	0.1000	0.2000	0.41	0.29
1796	30,000	700	35,000	1,000	0.0233	0.0286	0.36	-0.15
1796	20,000	3,500	24,000	2,200	0.1750	0.0917	-0.46	-0.18
1797	11,000	1,200	8,000	1,000	0.1091	0.1250	-0.18	0.32
1797	43,000	500	24,000	700	0.0116	0.0292	0.34	0.58
1797	40,000	3,800	20,000	4,000	0.0950	0.2000	0.05	0.69
1797	22,000	2,200	28,000	4,000	0.1000	0.1429	0.60	-0.24
1797	18,000	200	17,000	300	0.0111	0.0176	0.41	0.06
1797	9,000	100	7,000	800	0.0111	0.1143	2.08	0.25
1797	28,000	1,200	16,000	1,300	0.0429	0.0813	0.08	0.56
1797	52,000	3,000	34,000	2,700	0.0577	0.0794	-0.11	0.42
1797	45,000	2,000	30,000	1,000	0.0444	0.0333	-0.69	0.41
1798	10,000	1,500	12,000	3,000	0.1500	0.2500	0.69	-0.18
1798	25,000	2,000	25,000	15,000	0.0800	0.6000	2.01	0.00
1798	20,000	300	6,000	2,000	0.0150	0.3333	1.90	1.20
1798	10,000	500	26,000	2,500	0.0500	0.0962	1.61	-0.96
1798	8,000	400	4,000	500	0.0500	0.1250	0.22	0.69
1798	3,500	200	4,500	4,000	0.0571	0.8889	3.00	-0.25
1798	3,000	500	12,000	1,000	0.1667	0.0833	0.69	-1.39
1798	1,100	200	5,000	400	0.1818	0.0800	0.69	-1.51
1799	25,000	400	6,000	750	0.0160	0.1250	0.63	1.43
1799	11,000	150	8,000	500	0.0136	0.0625	1.20	0.32
1799	13,000	200	4,000	200	0.0154	0.0500	0.00	1.18
1799	8,000	200	5,000	300	0.0250	0.0600	0.41	0.47
1799	15,000	400	7,000	400	0.0267	0.0571	0.00	0.76
1799	15,000	400	15,000	1,200	0.0267	0.0800	1.10	0.00
1799	12,000	400	5,000	300	0.0333	0.0600	-0.29	0.88
1799	14,000	500	9,000	500	0.0357	0.0556	0.00	0.44
1799	11,000	400	12,000	1,000	0.0364	0.0833	0.92	-0.09
1799	8,000	400	8,000	600	0.0500	0.0750	0.41	0.00
1799	6,000	300	6,000	500	0.0500	0.0833	0.51	0.00
1799	12,000	600	10,000	1,000	0.0500	0.1000	0.51	0.18

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1799	3,200	320	7,000	450	0.1000	0.0643	0.34	-0.78
1799	9,600	100	3,400	3,000	0.0104	0.8824	3.40	1.04
1799	15,000	200	7,000	1,000	0.0133	0.1429	1.61	0.76
1799	46,000	4,000	41,000	3,500	0.0870	0.0854	-0.13	0.12
1799	7,000	100	5,000	1,200	0.0143	0.2400	2.48	0.34
1799	22,000	1,000	23,000	2,100	0.0455	0.0913	0.74	-0.04
1799	29,000	2,150	15,000	3,400	0.0741	0.2267	0.46	0.66
1799	20,000	400	7,500	2,000	0.0200	0.2667	1.61	0.98
1799	4,500	100	6,500	1,000	0.0222	0.1538	2.30	-0.37
1799	9,000	200	6,000	1,100	0.0222	0.1833	1.70	0.41
1799	12,000	300	2,200	400	0.0250	0.1818	0.29	1.70
1799	52,000	3,800	28,000	4,000	0.0731	0.1429	0.05	0.62
1799	6,500	200	7,000	1,000	0.0308	0.1429	1.61	-0.07
1799	11,000	400	3,100	400	0.0364	0.1290	0.00	1.27
1799	12,000	500	6,000	3,000	0.0417	0.5000	1.79	0.69
1799	4,300	200	3,300	400	0.0465	0.1212	0.69	0.26
1799	46,000	4,900	38,000	2,000	0.1065	0.0526	-0.90	0.19
1799	12,000	600	8,000	2,400	0.0500	0.3000	1.39	0.41
1799	11,500	600	10,000	1,500	0.0522	0.1500	0.92	0.14
1799	50,000	7,000	35,000	7,000	0.1400	0.2000	0.00	0.36
1799	8,000	500	11,000	1,500	0.0625	0.1364	1.10	-0.32
1799	8,000	500	4,400	2,200	0.0625	0.5000	1.48	0.60
1799	15,000	1,000	7,000	800	0.0667	0.1143	-0.22	0.76
1799	14,000	1,000	8,000	1,000	0.0714	0.1250	0.00	0.56
1799	6,000	500	4,000	1,000	0.0833	0.2500	0.69	0.41
1799	55,000	2,200	45,000	1,300	0.0400	0.0289	-0.53	0.20
1799	22,300	700	9,500	2,000	0.0314	0.2105	1.05	0.85
1799	1,000	100	8,000	4,000	0.1000	0.5000	3.69	-2.08
1799	30,000	1,800	20,000	3,000	0.0600	0.1500	0.51	0.41
1799	16,000	500	17,000	700	0.0313	0.0412	0.34	-0.06
1799	18,000	2,000	8,500	1,000	0.1111	0.1176	-0.69	0.75
1799	7,500	900	12,000	3,000	0.1200	0.2500	1.20	-0.47
1799	4,000	500	26,000	6,000	0.1250	0.2308	2.48	-1.87
1799	23,000	800	10,000	2,200	0.0348	0.2200	1.01	0.83
1799	32,000	2,100	13,000	3,100	0.0656	0.2385	0.39	0.90
1799	24,000	1,200	21,000	1,600	0.0500	0.0762	0.29	0.13
1799	3,000	500	9,000	1,500	0.1667	0.1667	1.10	-1.10
1799	6,000	1,100	18,000	12,000	0.1833	0.6667	2.39	-1.10
1799	21,000	6,000	9,000	2,000	0.2857	0.2222	-1.10	0.85
1799	5,000	2,000	12,000	4,000	0.4000	0.3333	0.69	-0.88

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1799	16,400	1,600	14,500	1,500	0.0976	0.1034	-0.06	0.12
1799	22,400	1,000	8,800	2,000	0.0446	0.2273	0.69	0.93
1799	33,500	4,000	23,000	6,000	0.1194	0.2609	0.41	0.38
1799	50,000	1,550	28,000	4,000	0.0310	0.1429	0.95	0.58
1799	37,000	5,000	33,000	9,500	0.1351	0.2879	0.64	0.11
1799	17,000	200	21,000	2,000	0.0118	0.0952	2.30	-0.21
1799	19,000	100	8,000	300	0.0053	0.0375	1.10	0.86
1800	66,000	4,000	50,000	4,100	0.0606	0.0820	0.02	0.28
1800	17,000	700	30,000	800	0.0412	0.0267	0.13	-0.57
1800	20,000	1,000	40,000	500	0.0500	0.0125	-0.69	-0.69
1800	25,000	2,000	20,000	1,250	0.0800	0.0625	-0.47	0.22
1800	5,000	200	3,000	400	0.0400	0.1333	0.69	0.51
1800	55,000	2,500	57,000	5,500	0.0455	0.0965	0.79	-0.04
1800	20,000	1,000	7,000	2,000	0.0500	0.2857	0.69	1.05
1800	52,000	3,000	48,000	2,400	0.0577	0.0500	-0.22	0.08
1800	10,000	600	3,500	1,000	0.0600	0.2857	0.51	1.05
1800	24,000	2,500	12,000	4,000	0.1042	0.3333	0.47	0.69
1800	14,000	1,000	7,000	2,000	0.0714	0.2857	0.69	0.69
1800	4,000	300	4,500	500	0.0750	0.1111	0.51	-0.12
1800	6,000	500	4,000	600	0.0833	0.1500	0.18	0.41
1800	6,000	500	3,000	1,450	0.0833	0.4833	1.06	0.69
1800	7,000	650	5,000	1,000	0.0929	0.2000	0.43	0.34
1800	8,000	1,000	10,000	950	0.1250	0.0950	-0.05	-0.22
1800	9,000	1,000	17,000	2,000	0.1111	0.1176	0.69	-0.64
1800	7,000	800	5,000	1,000	0.1143	0.2000	0.22	0.34
1800	40,000	1,000	10,000	1,000	0.0250	0.1000	0.00	1.39
1800	12,000	3,000	16,000	2,100	0.2500	0.1313	-0.36	-0.29
1800	28,000	6,500	31,000	7,000	0.2321	0.2258	0.07	-0.10
1800	8,000	200	5,000	200	0.0250	0.0400	0.00	0.47
1800	5,000	200	7,000	250	0.0400	0.0357	0.22	-0.34
1800	84,000	3,000	72,000	3,000	0.0357	0.0417	0.00	0.15
1800	37,000	2,000	35,000	1,200	0.0541	0.0343	-0.51	0.06
1800	11,000	800	10,000	700	0.0727	0.0700	-0.13	0.10
1800	6,000	500	4,000	400	0.0833	0.1000	-0.22	0.41
1800	6,000	500	3,500	1,100	0.0833	0.3143	0.79	0.54
1801	15,000	1,300	5,000	700	0.0867	0.1400	-0.62	1.10
1801	5,500	650	2,000	350	0.1182	0.1750	-0.62	1.01
1801	12,000	1,500	10,000	3,000	0.1250	0.3000	0.69	0.18
1805	11,000	800	7,000	1,000	0.0727	0.1429	0.22	0.45
1805	65,000	10,000	83,000	16,000	0.1538	0.1928	0.47	-0.24

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1805	49,000	5,700	46,000	6,300	0.1163	0.1370	0.10	0.06
1805	16,000	1,000	8,000	4,000	0.0625	0.5000	1.39	0.69
1805	25,000	4,000	8,000	5,000	0.1600	0.6250	0.22	1.14
1805	20,000	700	15,000	800	0.0350	0.0533	0.13	0.29
1805	12,000	600	4,000	400	0.0500	0.1000	-0.41	1.10
1805	9,000	600	4,800	400	0.0667	0.0833	-0.41	0.63
1805	7,000	500	8,000	400	0.0714	0.0500	-0.22	-0.13
1805	25,000	1,100	6,000	600	0.0440	0.1000	-0.61	1.43
1805	30,000	2,000	7,000	1,200	0.0667	0.1714	-0.51	1.46
1805	13,000	400	3,000	1,500	0.0308	0.5000	1.32	1.47
1806	27,000	1,000	22,000	600	0.0370	0.0273	-0.51	0.20
1806	12,000	200	9,000	700	0.0167	0.0778	1.25	0.29
1806	27,300	7,100	50,000	10,000	0.2601	0.2000	0.34	-0.61
1806	16,000	800	13,700	1,000	0.0500	0.0730	0.22	0.16
1806	12,000	1,000	11,000	1,000	0.0833	0.0909	0.00	0.09
1806	96,000	6,000	54,000	12,000	0.0625	0.2222	0.69	0.58
1806	18,000	850	5,500	1,400	0.0472	0.2545	0.50	1.19
1806	26,000	3,300	44,000	3,500	0.1269	0.0795	0.06	-0.53
1806	4,800	350	4,300	1,700	0.0729	0.3953	1.58	0.11
1806	30,000	2,000	15,000	2,000	0.0667	0.1333	0.00	0.69
1807	6,000	3,000	14,000	5,000	0.5000	0.3571	0.51	-0.85
1807	45,000	6,000	16,000	3,000	0.1333	0.1875	-0.69	1.03
1807	87,000	12,000	61,000	20,000	0.1379	0.3279	0.51	0.36
1807	75,000	23,000	83,000	23,000	0.3067	0.2771	0.00	-0.10
1807	12,000	700	9,000	1,100	0.0583	0.1222	0.45	0.29
1807	20,000	1,200	18,000	2,500	0.0600	0.1389	0.73	0.11
1807	1,000	150	7,000	2,000	0.1500	0.2857	2.59	-1.95
1807	12,000	2,800	6,000	1,500	0.2333	0.2500	-0.62	0.69
1807	65,000	12,500	95,000	9,000	0.1923	0.0947	-0.33	-0.38
1807	63,000	2,500	17,000	1,800	0.0397	0.1059	-0.33	1.31
1808	24,500	200	11,000	2,500	0.0082	0.2273	2.53	0.80
1808	32,000	1,000	22,000	3,000	0.0313	0.1364	1.10	0.37
1808	35,000	700	45,000	3,000	0.0200	0.0667	1.46	-0.25
1808	21,000	300	19,000	400	0.0143	0.0211	0.29	0.10
1808	13,000	3,000	15,500	3,500	0.2308	0.2258	0.15	-0.18
1808	19,000	750	13,000	1,500	0.0395	0.1154	0.69	0.38
1808	14,000	400	15,000	1,000	0.0286	0.0667	0.92	-0.07
1808	50,000	6,000	30,000	18,000	0.1200	0.6000	1.10	0.51
1808	8,000	300	12,000	200	0.0375	0.0167	-0.41	-0.41
1808	15,000	1,100	22,000	1,000	0.0733	0.0455	-0.10	-0.38

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1808	15,000	500	4,400	600	0.0333	0.1364	0.18	1.23
1808	15,000	600	9,000	1,000	0.0400	0.1111	0.51	0.51
1808	25,000	1,000	3,500	1,500	0.0400	0.4286	0.41	1.97
1808	7,000	300	3,000	2,000	0.0429	0.6667	1.90	0.85
1808	21,000	1,100	23,000	900	0.0524	0.0391	-0.20	-0.09
1809	5,000	400	22,000	2,000	0.0800	0.0909	1.61	-1.48
1809	45,000	3,000	25,000	1,900	0.0667	0.0760	-0.46	0.59
1809	18,000	200	10,000	1,700	0.0111	0.1700	2.14	0.59
1809	54,000	6,000	47,000	7,100	0.1111	0.1511	0.17	0.14
1809	13,000	200	12,000	2,000	0.0154	0.1667	2.30	0.08
1809	11,000	200	6,300	1,100	0.0182	0.1746	1.70	0.56
1809	15,000	300	5,000	600	0.0200	0.1200	0.69	1.10
1809	15,000	900	20,000	1,500	0.0600	0.0750	0.51	-0.29
1809	33,000	2,000	50,000	4,000	0.0606	0.0800	0.69	-0.42
1809	10,000	400	8,000	1,000	0.0400	0.1250	0.92	0.22
1809	45,000	4,000	40,000	3,000	0.0889	0.0750	-0.29	0.12
1809	16,000	400	23,000	4,000	0.0250	0.1739	2.30	-0.36
1809	35,000	4,000	37,000	3,500	0.1143	0.0946	-0.13	-0.06
1809	15,000	650	3,000	1,000	0.0433	0.3333	0.43	1.61
1809	39,000	3,600	41,000	3,000	0.0923	0.0732	-0.18	-0.05
1809	12,000	600	18,000	2,000	0.0500	0.1111	1.20	-0.41
1809	17,500	1,000	23,000	8,000	0.0571	0.3478	2.08	-0.27
1809	36,000	2,000	58,000	450	0.0556	0.0078	-1.49	-0.48
1809	60,000	1,000	46,000	2,800	0.0167	0.0609	1.03	0.27
1809	27,000	150	13,000	600	0.0056	0.0462	1.39	0.73
1809	1,500	100	2,300	500	0.0667	0.2174	1.61	-0.43
1809	7,000	500	9,000	1,500	0.0714	0.1667	1.10	-0.25
1809	13,300	1,000	11,700	1,500	0.0752	0.1282	0.41	0.13
1809	34,000	15,000	9,400	5,200	0.4412	0.5532	-1.06	1.29
1809	66,000	3,000	74,000	6,000	0.0455	0.0811	0.69	-0.11
1809	74,000	4,000	60,000	3,200	0.0541	0.0533	-0.22	0.21
1809	30,000	4,600	30,000	2,400	0.1533	0.0800	-0.65	0.00
1809	12,000	1,600	9,000	700	0.1333	0.0778	-0.83	0.29
1809	11,300	800	20,000	4,000	0.0708	0.2000	1.61	-0.57
1809	21,000	700	11,000	13,000	[illegible]	1.1818	2.92	0.65
1809	5,000	600	18,000	3,000	[illegible]	0.1667	1.61	-1.28
1809	40,000	300	39,000	500	[illegible]	0.0128	0.51	0.03
1809	11,000	1,700	5,000	1,000	0.1545	0.2000	-0.53	0.79
1809	21,000	2,400	23,000	3,300	0.1143	0.1435	0.32	-0.09
1809	27,000	800	18,000	1,500	0.0296	0.0833	0.63	0.41

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1809	52,000	1,000	58,000	2,750	0.0192	0.0474	1.01	-0.11
1809	160,000	30,000	130,000	19,000	0.1875	0.1462	-0.46	0.21
1809	99,000	21,500	66,000	23,000	0.2172	0.3485	0.07	0.41
1809	12,700	100	12,500	500	0.0079	0.0400	1.61	0.02
1809	15,500	200	11,300	1,000	0.0129	0.0885	1.61	0.32
1809	1,500	40	3,500	85	0.0267	0.0243	0.75	-0.85
1809	18,000	500	12,000	1,000	0.0278	0.0833	0.69	0.41
1809	9,000	300	8,000	700	0.0333	0.0875	0.85	0.12
1809	72,000	2,000	78,000	2,000	0.0278	0.0256	0.00	-0.08
1809	26,000	400	14,000	1,350	0.0154	0.0964	1.22	0.62
1809	16,000	2,000	30,000	8,000	0.1250	0.2667	1.39	-0.63
1809	30,000	2,000	15,000	1,000	0.0667	0.0667	-0.69	0.69
1810	25,000	12,000	15,000	6,000	0.4800	0.4000	-0.69	0.51
1810	70,000	1,500	5,500	1,500	0.0214	0.2727	0.00	2.54
1810	27,000	800	2,800	200	0.0296	0.0714	-1.39	2.27
1810	32,000	1,300	58,000	4,500	0.0406	0.0776	1.24	-0.59
1810	22,000	2,000	35,000	5,000	0.0909	0.1429	0.92	-0.46
1810	30,000	1,600	20,000	1,800	0.0533	0.0900	0.12	0.41
1810	6,000	100	1,800	400	0.0167	0.2222	1.39	1.20
1810	6,000	100	7,300	500	0.0167	0.0685	1.61	-0.20
1810	19,000	1,400	30,000	3,000	0.0737	0.1000	0.76	-0.46
1810	23,000	1,600	5,000	3,000	0.0696	0.6000	0.63	1.53
1810	60,000	700	6,000	1,000	0.0117	0.1667	0.36	2.30
18[illegible]	13,000	1,000	9,000	1,700	0.0769	0.1889	0.53	0.37
1810	12,000	1,000	11,000	1,500	0.0833	0.1364	0.41	0.09
1811	19,500	1,000	3,000	400	0.0513	0.1333	-0.92	1.87
1811	35,000	1,500	45,000	2,700	0.0429	0.0600	0.59	-0.25
1811	8,000	1,000	10,000	4,000	0.1250	0.4000	1.39	-0.22
1811	15,000	2,000	7,000	2,000	0.1333	0.2857	0.00	0.76
1811	12,000	2,000	10,000	2,500	0.1667	0.2500	0.22	0.18
1811	6,000	1,000	6,000	2,100	0.1667	0.3500	0.74	0.00
1811	20,000	800	60,000	1,500	0.0400	0.0250	0.63	-1.10
1811	4,500	150	10,400	600	0.0333	0.0577	1.39	-0.84
1811	17,000	1,500	21,000	7,000	0.0882	0.3333	1.54	-0.21
1811	4,000	300	11,000	900	0.0750	0.0818	1.10	-1.01
1811	6,000	500	12,000	1,000	0.0833	0.0833	0.69	-0.69
1811	32,000	7,000	18,000	8,000	0.2188	0.4444	0.13	0.58
1811	18,000	4,300	18,000	8,000	0.2389	0.4444	0.62	0.00
1811	17,000	1,000	23,000	1,000	0.0588	0.0435	0.00	-0.30
1811	5,000	100	3,000	2,000	0.0200	0.6667	3.00	0.51

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1811	33,000	2,000	33,000	2,000	0.0606	0.0606	0.00	0.00
1811	18,000	600	22,000	1,100	0.0333	0.0500	0.61	-0.20
1811	10,000	500	12,000	1,500	0.0500	0.1250	1.10	-0.18
1812	30,000	400	30,000	400	0.0133	0.0133	0.00	0.00
1812	4,000	200	10,000	3,000	0.0500	0.3000	2.71	-0.92
1812	6,000	300	1,500	800	0.0500	0.5333	0.98	1.39
1812	3,500	200	2,000	250	0.0571	0.1250	0.22	0.56
1812	51,000	1,850	5,000	1,300	0.0363	0.2600	-0.35	2.32
1812	23,000	4,300	20,000	3,700	0.1870	0.1850	-0.15	0.14
1812	28,000	3,400	28,000	2,700	0.1214	0.0964	-0.23	0.00
1812	35,000	1,300	27,000	1,500	0.0371	0.0556	0.14	0.26
1812	20,000	500	7,000	700	0.0250	0.1000	0.34	1.05
1812	32,000	10,000	27,000	6,000	0.3125	0.2222	-0.51	0.17
1812	9,000	800	10,000	1,200	0.0889	0.1200	0.41	-0.11
1812	33,000	10,000	87,000	8,000	0.3030	0.0920	-0.22	-0.97
1812	90,000	2,000	50,000	6,000	0.0222	0.1200	1.10	0.59
1812	46,000	5,200	42,000	10,000	0.1130	0.2381	0.65	0.09
1812	24,000	6,000	24,000	8,000	0.2500	0.3333	0.29	0.00
1812	124,000	28,000	122,000	43,000	0.2258	0.3525	0.43	0.02
1812	36,000	4,000	20,000	6,000	0.1111	0.3000	0.41	0.59
1812	34,000	6,000	22,000	6,000	0.1765	0.2727	0.00	0.44
1812	40,000	250	1,800	800	0.0063	0.4444	1.16	3.10
1812	180,000	10,000	120,000	6,000	0.0556	0.0500	-0.51	0.41
1812	36,000	800	20,000	2,000	0.0222	0.1000	0.92	0.59
1812	35,000	8,800	25,000	6,000	0.2514	0.2400	-0.38	0.34
1812	27,000	1,800	25,000	4,000	0.0667	0.1600	0.80	0.08
1812	2,000	200	32,000	550	0.1000	0.0172	1.01	-2.77
1812	50,000	600	700	200	0.0120	0.2857	-1.10	4.27
1812	33,000	600	2,600	300	0.0182	0.1154	-0.69	2.54
1812	4,500	50	3,500	500	0.0111	0.1429	2.30	0.25
1812	6,600	100	6,000	300	0.0152	0.0500	1.10	0.10
1812	16,000	900	22,000	1,500	0.0563	0.0682	0.51	-0.32
1812	8,000	200	10,000	1,200	0.0250	0.1200	1.79	-0.22
1812	28,000	3,000	25,000	2,200	0.1071	0.0880	-0.31	0.11
1812	17,000	900	16,000	1,900	0.0529	0.1188	0.75	0.06
1813	80,000	9,000	70,000	6,500	0.1125	0.0929	-0.33	0.13
1813	70,000	4,600	50,000	5,400	0.0657	0.1080	0.16	0.34
1813	144,000	19,200	93,000	12,000	0.1333	0.1290	-0.47	0.44
1813	15,000	700	12,000	2,000	0.0467	0.1667	1.05	0.22
1813	160,000	8,000	100,000	16,000	0.0500	0.1600	0.69	0.47

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1813	16,000	1,900	15,000	900	0.1188	0.0600	-0.75	0.06
1813	10,000	500	4,000	1,100	0.0500	0.2750	0.79	0.92
1813	4,800	250	2,600	300	0.0521	0.1154	0.18	0.61
1813	20,000	1,100	8,500	2,100	0.0550	0.2471	0.65	0.86
1813	10,000	600	4,000	500	0.0600	0.1250	-0.18	0.92
1813	12,000	1,100	4,500	600	0.0917	0.1333	-0.61	0.98
1813	32,000	1,500	18,000	1,700	0.0469	0.0944	0.13	0.58
1813	3,000	250	2,500	500	0.0833	0.2000	0.69	0.18
1813	20,000	2,000	4,000	1,400	0.1000	0.3500	-0.36	1.61
1813	40,000	3,000	20,000	6,000	0.0750	0.3000	0.69	0.69
1813	19,000	2,000	3,500	1,350	0.1053	0.3857	-0.39	1.69
1813	24,000	600	50,000	1,200	0.0250	0.0240	0.69	-0.73
1813	24,000	1,000	18,000	1,500	0.0417	0.0833	0.41	0.29
1813	80,000	4,000	60,000	12,000	0.0500	0.2000	1.10	0.29
1813	4,000	500	7,000	1,000	0.1250	0.1429	0.69	-0.56
1813	21,400	800	18,000	1,500	0.0374	0.0833	0.63	0.17
1813	32,000	1,000	38,000	3,600	0.0313	0.0947	1.28	-0.17
1813	13,400	1,800	7,400	1,200	0.1343	0.1622	-0.41	0.59
1813	12,000	1,800	11,000	2,000	0.1500	0.1818	0.11	0.09
1813	17,000	700	15,000	100	0.0412	0.0067	-1.95	0.13
1813	90,000	5,300	50,000	3,200	0.0589	0.0640	-0.50	0.59
1813	60,000	6,000	40,000	5,000	0.1000	0.1250	-0.18	0.41
1813	37,000	6,000	45,000	4,000	0.1622	0.0889	-0.41	-0.20
1813	17,000	300	20,000	2,000	0.0176	0.1000	1.90	-0.16
1813	42,000	1,600	32,000	1,100	0.0381	0.0344	-0.37	0.27
1813	90,000	4,900	60,000	6,000	0.0544	0.1000	0.20	0.41
1813	90,000	5,000	4,800	1,900	0.0556	0.3958	-0.97	2.93
1813	60,000	2,500	50,000	1,500	0.0417	0.0300	-0.51	0.18
1813	8,000	100	4,000	1,700	0.0125	0.4250	2.83	0.69
1813	55,000	8,000	60,000	11,000	0.1455	0.1833	0.32	-0.09
1813	100,000	10,000	200,000	15,000	0.1000	0.0750	0.41	-0.69
1813	15,400	250	5,000	1,400	0.0162	0.2800	1.72	1.12
1813	160,000	2,000	100,000	3,000	0.0125	0.0300	0.41	0.47
1813	167,000	21,200	97,000	11,000	0.1269	0.1134	-0.66	0.54
1813	325,000	75,000	175,000	45,000	0.2308	0.2571	-0.51	0.62
1813	103,000	11,000	37,000	9,000	0.1068	0.2432	-0.20	1.02
1814	15,000	300	4,000	400	0.0200	0.1000	0.29	1.32
1814	40,000	5,600	100,000	5,000	0.1400	0.0500	-0.11	-0.92
1814	8,000	450	5,000	800	0.0563	0.1600	0.58	0.47
1814	36,000	3,000	30,000	3,000	0.0833	0.1000	0.00	0.18

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1814	10,000	600	8,800	1,500	0.0600	0.1705	0.92	0.13
1814	25,000	2,100	39,000	3,000	0.0840	0.0769	0.36	-0.44
1814	7,000	500	4,000	500	0.0714	0.1250	0.00	0.56
1814	100,000	9,000	42,000	7,000	0.0900	0.1667	-0.25	0.87
1814	34,000	3,000	32,000	2,800	0.0882	0.0875	-0.07	0.06
1814	12,000	500	10,000	300	0.0417	0.0300	-0.51	0.18
1814	11,000	1,000	16,000	5,000	0.0909	0.3125	1.61	-0.37
1814	6,000	650	11,000	1,000	0.1083	0.0909	0.43	-0.61
1814	15,000	400	10,000	800	0.0267	0.0800	0.69	0.41
1814	30,000	1,900	20,000	800	0.0633	0.0400	-0.86	0.41
1814	100,000	3,000	30,000	3,400	0.0300	0.1133	0.13	1.20
1814	5,000	600	4,000	2,400	0.1200	0.6000	1.39	0.22
1814	90,000	1,600	25,000	2,600	0.0178	0.1040	0.49	1.28
1814	5,000	500	7,000	300	0.1000	0.0429	-0.51	-0.34
1814	50,000	1,000	20,000	1,000	0.0200	0.0500	0.00	0.92
1814	28,000	2,000	21,000	5,000	0.0714	0.2381	0.92	0.29
1814	60,000	6,700	32,000	4,000	0.1117	0.1250	-0.52	0.63
1814	123,000	6,000	41,000	3,000	0.0488	0.0732	-0.69	1.10
1814	2,700	500	9,000	1,900	0.1852	0.2111	1.34	-1.20
1814	4,000	770	5,000	650	0.1925	0.1300	-0.17	-0.22
1814	25,000	1,400	13,000	500	0.0560	0.0385	-1.03	0.65
1814	20,000	500	30,000	3,000	0.0250	0.1000	1.79	-0.41
1814	45,000	3,200	35,000	3,000	0.0711	0.0857	-0.06	0.25
1814	30,000	800	13,000	2,300	0.0267	0.1769	1.06	0.84
1814	12,000	400	4,000	600	0.0333	0.1500	0.41	1.10
1814	18,000	600	9,000	2,400	0.0333	0.2667	1.39	0.69
1814	14,000	1,600	25,000	1,700	0.1143	0.0680	0.06	-0.58
1814	13,000	500	9,000	1,000	0.0385	0.1111	0.69	0.37
1814	30,000	2,000	15,000	1,400	0.0667	0.0933	-0.36	0.69
1814	100,000	3,500	50,000	4,000	0.0350	0.0800	0.13	0.69
1815	20,000	1,000	17,000	500	0.0500	0.0294	-0.69	0.16
1815	120,000	19,000	72,000	25,000	0.1583	0.3472	0.27	0.51
1815	10,000	600	5,000	1,600	0.0600	0.3200	0.98	0.69
1815	71,000	12,000	84,000	16,000	0.1690	0.1905	0.29	-0.17
1815	32,000	5,100	21,000	4,400	0.1594	0.2095	-0.15	0.42
1815	40,000	2,000	20,000	1,000	0.0500	0.0500	-0.69	0.69
1815	32,000	2,500	24,000	2,500	0.0781	0.1042	0.00	0.29
1815	11,000	700	29,000	1,700	0.0636	0.0586	0.89	-0.97
1815	6,000	20	12,000	2,000	0.0033	0.1667	4.61	-0.69
1816	5,000	800	12,000	3,000	0.1600	0.2500	1.32	-0.88

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1828	24,000	2,000	30,000	4,000	0.0833	0.1333	0.69	-0.22
1828	20,000	6,000	15,000	8,000	0.3000	0.5333	0.29	0.29
1828	20,000	1,600	6,000	1,400	0.0800	0.2333	-0.13	1.20
1828	7,000	600	11,000	3,500	0.0857	0.3182	1.76	-0.45
1828	4,500	1,500	35,000	2,500	0.3333	0.0714	0.51	-2.05
1828	10,000	1,800	30,000	5,000	0.1800	0.1667	1.02	-1.10
1828	7,000	1,000	20,000	1,000	0.1429	0.0500	0.00	-1.05
1828	11,000	500	20,000	1,500	0.0455	0.0750	1.10	-0.60
1828	17,000	3,000	8,000	1,500	0.1765	0.1875	-0.69	0.75
1829	19,000	100	35,000	1,000	0.0053	0.0286	2.30	-0.61
1829	28,000	2,500	40,000	3,000	0.0893	0.0750	0.18	-0.36
1829	18,000	100	20,000	500	0.0056	0.0250	1.61	-0.11
1829	37,000	3,000	15,000	6,000	0.0811	0.4000	0.69	0.90
1831	38,000	4,500	32,000	7,000	0.1184	0.2188	0.44	0.17
1831	52,000	3,800	44,000	2,900	0.0731	0.0659	-0.27	0.17
1831	50,000	500	26,000	1,000	0.0100	0.0385	0.69	0.65
1831	78,000	10,600	37,000	7,800	0.1359	0.2108	-0.31	0.75
1831	22,000	1,500	24,000	1,700	0.0682	0.0708	0.13	-0.09
1831	80,000	10,000	50,000	9,000	0.1250	0.1800	-0.11	0.47
1832	22,000	2,000	44,000	3,000	0.0909	0.0682	0.41	-0.69
1832	15,000	500	20,000	2,000	0.0333	0.1000	1.39	-0.29
1832	14,000	500	17,000	2,500	0.0357	0.1471	1.61	-0.19
1839	43,000	500	33,000	1,000	0.0116	0.0303	0.69	0.26
1840	4,700	100	5,000	2,000	0.0213	0.4000	3.00	-0.06
1844	10,000	70	30,000	2,000	0.0070	0.0667	3.35	-1.10
1847	4,400	720	20,000	2,000	0.1636	0.1000	1.02	-1.51
1848	19,000	500	16,000	700	0.0263	0.0438	0.34	0.17
1848	55,000	900	75,000	900	0.0164	0.0120	0.00	-0.31
1848	16,000	500	11,000	800	0.0313	0.0727	0.47	0.37
1848	23,000	100	6,000	300	0.0043	0.0500	1.10	1.34
1848	19,000	650	4,000	700	0.0342	0.1750	0.07	1.56
1848	31,000	900	18,000	1,400	0.0290	0.0778	0.44	0.54
1848	25,000	500	5,000	200	0.0200	0.0400	-0.92	1.61
1848	16,000	100	6,000	1,000	0.0063	0.1667	2.30	0.98
1848	11,000	100	6,000	150	0.0091	0.0250	0.41	0.61
1848	18,000	300	11,000	400	0.0167	0.0364	0.29	0.49
1848	42,000	600	10,000	300	0.0143	0.0300	-0.69	1.44
1848	28,000	350	42,000	700	0.0125	0.0167	0.69	-0.41
1848	19,000	260	41,000	750	0.0137	0.0183	1.06	-0.77
1849	28,000	350	38,000	1,200	0.0125	0.0316	1.23	-0.31

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1849	9,000	1,000	8,000	2,000	0.1111	0.2500	0.69	0.12
1849	16,000	310	24,000	300	0.0194	0.0125	-0.03	-0.41
1849	10,000	200	6,000	1,000	0.0200	0.1667	1.61	0.51
1849	41,000	2,300	59,000	3,000	0.0561	0.0508	0.27	-0.36
1849	37,000	200	45,000	500	0.0054	0.0111	0.92	-0.20
1849	25,000	600	15,000	800	0.0240	0.0533	0.29	0.51
1849	11,000	100	8,000	1,100	0.0091	0.1375	2.40	0.32
1849	30,000	550	20,000	1,500	0.0183	0.0750	1.00	0.41
1849	20,000	200	8,000	2,000	0.0100	0.2500	2.30	0.92
1849	34,000	800	40,000	1,200	0.0235	0.0300	0.41	-0.16
1849	25,000	300	38,000	500	0.0120	0.0132	0.51	-0.42
1849	51,000	700	50,000	1,300	0.0137	0.0260	0.62	0.02
1849	15,000	250	8,000	800	0.0167	0.1000	1.16	0.63
1849	12,000	200	7,000	1,300	0.0167	0.1857	1.87	0.54
1849	53,000	1,200	37,000	2,000	0.0226	0.0541	0.51	0.36
1849	20,000	1,870	14,000	1,350	0.0935	0.0964	-0.33	0.36
1849	4,000	50	4,000	200	0.0125	0.0500	1.39	0.00
1850	3,300	450	11,500	700	0.1364	0.0609	0.44	-1.25
1850	4,000	250	12,000	300	0.0625	0.0250	0.18	-1.10
1850	39,000	3,200	26,000	1,750	0.0821	0.0673	-0.60	0.41
1853	3,000	1,000	7,000	1,000	0.3333	0.1429	0.00	-0.85
1854	6,000	2,000	10,000	3,600	0.3333	0.3600	0.59	-0.51
1854	20,000	800	60,000	1,600	0.0400	0.0267	0.69	-1.10
1854	21,000	600	29,000	1,200	0.0286	0.0414	0.69	-0.32
1854	65,000	4,400	35,000	5,000	0.0677	0.1429	0.13	0.62
1854	30,000	7,000	36,000	12,000	0.2333	0.3333	0.54	-0.18
1854	175,000	54,000	120,000	10,300	0.3086	0.0858	-1.66	0.38
1855	22,000	500	18,000	1,500	0.0227	0.0833	1.10	0.20
1855	50,000	1,800	74,000	7,300	0.0360	0.0986	1.40	-0.39
1855	30,000	7,000	16,000	3,000	0.2333	0.1875	-0.85	0.63
1856	19,000	1,000	70,000	3,000	0.0526	0.0429	1.10	-1.30
1859	48,000	3,900	62,000	5,700	0.0813	0.0919	0.38	-0.26
1859	40,500	1,000	8,500	360	0.0247	0.0424	-1.02	1.56
1859	8,300	800	18,700	1,100	0.0964	0.0588	0.32	-0.81
1859	14,300	1,460	130,000	13,100	0.1021	0.1008	2.19	-2.21
1859	26,000	1,000	18,000	1,700	0.0385	0.0944	0.53	0.37
1860	18,000	1,500	38,000	1,700	0.0833	0.0447	0.13	-0.75
1860	3,500	700	5,000	200	0.2000	0.0400	-1.25	-0.36
1860	30,000	200	6,000	500	0.0067	0.0833	0.92	1.61
1861	10,000	1,230	6,000	1,140	0.1230	0.1900	-0.08	0.51

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1861	32,000	2,000	35,000	1,600	0.0625	0.0457	-0.22	-0.09
1862	50,000	9,400	70,000	10,200	0.1880	0.1457	0.08	-0.34
1862	68,000	3,150	34,000	3,700	0.0463	0.1088	0.16	0.69
1862	94,000	19,000	106,000	10,000	0.2021	0.0943	-0.64	-0.12
1862	24,000	1,800	15,000	1,600	0.0750	0.1067	-0.12	0.47
1862	8,000	450	16,000	1,050	0.0563	0.0656	0.85	-0.69
1862	12,000	1,200	16,000	1,100	0.1000	0.0688	-0.09	-0.29
1862	27,000	2,660	17,000	2,000	0.0985	0.1176	-0.29	0.46
1862	14,000	1,000	11,000	1,000	0.0714	0.0909	0.00	0.24
1862	60,000	10,200	40,000	9,800	0.1700	0.2450	-0.04	0.41
1862	39,000	6,500	51,000	4,400	0.1667	0.0863	-0.39	-0.27
1862	20,000	1,300	18,000	1,800	0.0650	0.1000	0.33	0.11
1862	25,000	2,200	22,000	2,700	0.0880	0.1227	0.20	0.13
1862	78,000	4,700	117,000	11,000	0.0603	0.0940	0.85	-0.41
1862	85,000	11,700	50,000	9,400	0.1376	0.1880	-0.22	0.53
1862	45,000	1,900	30,000	1,300	0.0422	0.0433	-0.38	0.41
1863	70,000	15,800	57,000	11,500	0.2257	0.2018	-0.32	0.21
1863	90,000	17,700	68,000	15,300	0.1967	0.2250	-0.15	0.28
1863	18,000	300	9,000	500	0.0167	0.0556	0.51	0.69
1863	80,000	10,800	130,000	11,400	0.1350	0.0877	0.05	-0.49
1863	25,000	200	33,000	1,200	0.0080	0.0364	1.79	-0.28
1863	65,000	5,600	45,000	2,600	0.0862	0.0578	-0.77	0.37
1863	14,000	2,000	16,000	4,000	0.1429	0.2500	0.69	-0.13
1863	45,000	9,600	38,000	9,300	0.2133	0.2447	-0.03	0.17
1863	30,000	2,300	18,000	3,000	0.0767	0.1667	0.27	0.51
1863	26,000	1,100	5,000	200	0.0423	0.0400	-1.70	1.65
1863	75,000	9,000	28,000	7,000	0.1200	0.2500	-0.25	0.99
1864	70,000	4,000	40,000	8,500	0.0571	0.2125	0.75	0.56
1864	60,000	1,700	100,000	11,000	0.0283	0.1100	1.87	-0.51
1864	37,000	5,600	27,000	1,300	0.1514	0.0481	-1.46	0.32
1864	55,000	3,100	38,000	15,000	0.0564	0.3947	1.58	0.37
1864	118,000	14,300	62,000	7,000	0.1212	0.1129	-0.71	0.64
1864	93,000	4,000	52,000	1,500	0.0430	0.0288	-0.98	0.58
1864	120,000	39,000	67,000	40,000	0.3250	0.5970	0.03	0.58
1864	100,000	16,200	50,000	5,000	0.1620	0.1000	-1.18	0.69
1864	25,000	3,000	30,000	3,000	0.1200	0.1000	0.00	-0.18
1864	52,000	4,000	38,000	3,000	0.0769	0.0789	-0.29	0.31
1864	5,000	480	5,000	420	0.0960	0.0840	-0.13	0.00
1864	60,000	700	30,000	4,600	0.0117	0.1533	1.88	0.69
1864	43,000	4,400	13,000	2,000	0.1023	0.1538	-0.79	1.20

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1864	72,000	1,700	48,000	4,800	0.0236	0.1000	1.04	0.41
1864	20,000	400	11,000	700	0.0200	0.0636	0.56	0.60
1864	37,000	1,200	23,000	1,200	0.0324	0.0522	0.00	0.48
1864	40,000	4,100	18,000	1,900	0.1025	0.1056	-0.77	0.80
1864	80,000	2,400	40,000	3,000	0.0300	0.0750	0.22	0.69
1864	6,000	800	18,000	3,000	0.1333	0.1667	1.32	-1.10
1864	5,500	430	6,000	900	0.0782	0.1500	0.74	-0.09
1865	60,000	1,500	30,000	1,700	0.0250	0.0567	0.13	0.69
1866	41,000	350	29,000	920	0.0085	0.0317	0.97	0.35
1866	220,000	8,900	215,000	23,600	0.0405	0.1098	0.98	0.02
1866	26,000	1,600	44,000	2,900	0.0615	0.0659	0.59	-0.53
1866	3,000	110	9,200	700	0.0367	0.0761	1.85	-1.12
1866	24,000	400	20,000	1,330	0.0167	0.0665	1.20	0.18
1866	16,000	1,500	9,000	900	0.0938	0.1000	-0.51	0.58
1866	26,000	250	34,000	600	0.0096	0.0176	0.88	-0.27
1866	25,000	210	20,000	490	0.0084	0.0245	0.85	0.22
1866	24,000	1,200	31,000	3,700	0.0500	0.1194	1.13	-0.26
1866	27,000	3,600	32,000	1,300	0.1333	0.0406	-1.02	-0.17
1866	75,000	5,200	90,000	3,400	0.0693	0.0378	-0.42	-0.18
1866	27,000	720	22,000	1,100	0.0267	0.0500	0.42	0.20
1866	30,000	1,400	23,000	3,330	0.0467	0.1448	0.87	0.27
1866	25,000	820	22,000	750	0.0328	0.0341	-0.09	0.13
1866	18,000	950	14,000	750	0.0528	0.0536	-0.24	0.25
1866	10,000	130	23,000	370	0.0130	0.0161	1.05	-0.83
1866	14,000	200	7,000	510	0.0143	0.0729	0.94	0.69
1867	3,000	400	7,000	1,000	0.1333	0.1429	0.92	-0.85
1870	82,000	9,300	41,000	8,000	0.1134	0.1951	-0.15	0.69
1370	38,000	3,600	52,000	4,500	0.0947	0.0865	0.22	-0.31
1870	6,000	500	8,000	1,400	0.0833	0.1750	1.03	-0.29
1870	4,000	400	8,000	1,800	0.1000	0.2250	1.50	-0.69
1870	35,000	1,300	25,000	1,400	0.0371	0.0560	0.07	0.34
1870	240,000	10,000	400,000	16,000	0.0417	0.0400	0.47	-0.51
1870	30,000	3,500	60,000	5,000	0.1167	0.0833	0.36	-0.69
1870	200,000	6,000	180,000	6,000	0.0300	0.0333	0.00	0.11
1870	40,000	1,300	36,000	1,200	0.0325	0.0333	-0.08	0.11
1870	187,000	19,700	113,000	12,800	0.1053	0.1133	-0.43	0.50
1870	35,000	500	30,000	700	0.0143	0.0233	0.34	0.15
1870	58,000	4,800	84,000	3,200	0.0828	0.0381	-0.41	-0.37
1870	25,000	1,700	65,000	5,000	0.0680	0.0769	1.08	-0.96
1870	14,000	500	8,000	800	0.0357	0.1000	0.47	0.56

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Ro	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1870	40,000	1,000	23,000	2,500	0.0250	0.1087	0.92	0.55
1870	9,000	500	20,000	2,000	0.0556	0.1000	1.39	-0.80
1870	6,000	500	10,000	900	0.0833	0.0900	0.59	-0.51
1870	200,000	8,300	120,000	17,000	0.0415	0.1417	0.72	0.51
1870	63,000	14,900	113,000	17,000	0.2365	0.1504	0.13	-0.58
1870	68,000	3,400	59,000	5,500	0.0500	0.0932	0.48	0.14
1870	26,000	1,000	41,000	1,100	0.0385	0.0268	0.10	-0.46
1870	30,000	1,000	20,000	700	0.0333	0.0350	-0.36	0.41
1870	40,000	1,000	60,000	2,200	0.0250	0.0367	0.79	-0.41
1870	78,000	2,800	96,000	3,600	0.0359	0.0375	0.25	-0.21
1870	86,000	2,000	64,000	3,000	0.0233	0.0469	0.41	0.30
1870	60,000	1,500	20,000	1,000	0.0250	0.0500	-0.41	1.10
1870	16,000	1,000	11,000	1,000	0.0625	0.0909	0.00	0.37
1870	26,000	2,200	18,000	4,800	0.0846	0.2667	0.78	0.37
1870	40,000	3,500	90,000	3,500	0.0875	0.0389	0.00	-0.81
1870	35,000	4,500	28,000	4,100	0.1286	0.1464	-0.09	0.22
1870	51,000	1,550	6,000	1,100	0.0304	0.1833	-0.34	2.14
1871	40,000	1,700	6,300	700	0.0425	0.1111	-0.89	1.85
1871	45,000	1,800	135,000	4,000	0.0400	0.0296	0.80	-1.10
1871	72,000	3,500	88,000	6,000	0.0486	0.0682	0.54	-0.20
1871	33,000	2,500	47,000	3,500	0.0758	0.0745	0.34	-0.35
1871	23,000	1,000	37,000	1,350	0.0435	0.0365	0.30	-0.48
1871	23,000	700	83,000	4,000	0.0304	0.0482	1.74	-1.28
1874	22,000	2,500	32,000	4,000	0.1136	0.1250	0.47	-0.37
1877	25,000	3,500	4,000	1,500	0.1400	0.3750	-0.85	1.83
1877	55,000	2,000	30,000	3,000	0.0364	0.1000	0.41	0.61
1877	29,000	1,700	13,000	2,000	0.0586	0.1538	0.16	0.80
1877	20,000	1,300	5,000	1,900	0.0650	0.3800	0.38	1.39
1877	12,000	2,000	30,000	5,000	0.1667	0.1667	0.92	-0.92
1877	40,000	5,000	60,000	4,000	0.1250	0.0667	-0.22	-0.41
1877	12,000	800	5,000	1,500	0.0667	0.3000	0.63	0.88
1877	11,000	600	16,000	1,000	0.0545	0.0625	0.51	-0.37
1877	17,000	1,000	11,000	1,400	0.0588	0.1273	0.34	0.44
1877	15,000	2,000	10,000	1,600	0.1333	0.1600	-0.22	0.41
1877	35,000	7,000	95,000	16,000	0.2000	0.1684	0.83	-1.00
1877	50,000	600	10,000	1,500	0.0120	0.1500	0.92	1.61
1877	24,000	1,600	8,000	2,400	0.0667	0.3000	0.41	1.10
1877	36,000	3,000	18,000	5,000	0.0833	0.2778	0.51	0.69
1877	70,000	800	35,000	3,000	0.0114	0.0857	1.32	0.69
1877	18,000	1,100	19,000	1,300	0.0611	0.0684	0.17	-0.05

APPENDIX C - HISTORICAL DATA SET

YEAR	RED INITIAL Ro	RED CASUALTY Rc	BLUE INITIAL Bo	BLUE CASUALTY Bc	Rc/Bo	Bc/Bo	Log RATIO Bc/Rc	Log RATIO Ro/Bo
1877	30,000	460	36,000	1,000	0.0153	0.0278	0.78	-0.18
1877	14,000	2,000	10,000	3,000	0.1429	0.3000	0.41	0.34
1877	22,000	1,000	32,000	1,400	0.0455	0.0438	0.34	-0.37
1877	20,000	3,500	33,000	7,500	0.1750	0.2273	0 76	-0.50
1877	10,000	200	7,000	1,000	0.0200	0.1429	1.61	0.36
1877	26,000	1,000	15,000	3,000	0.0385	0.2000	1.10	0.55
1877	18,000	5,400	27,000	7,000	0.3000	0.2593	0.26	-0.41
1878	18,000	300	10,000	2,400	0.0167	0.2400	2.08	0.59
1878	67,000	6,000	29,000	4,500	0.0896	0.1552	-0.29	0.84
1885	50,000	1,900	32,000	1,100	0.0380	0.0344	-0.55	0.45
1885	32,000	2,500	30,000	5,000	0.0781	0.1667	0.69	0.06
1889	60,000	5,000	8,000	2,200	0.0833	0.2750	-0.82	2.01
1891	11,000	1,600	14,000	3,400	0.1455	0.2429	0.75	-0.24
1894	12,000	700	15,000	3,500	0.0583	0.2333	1.61	-0.22
1894	6,000	600	5,000	500	0.1000	0.1000	-0.18	0.18
1897	47,000	1,800	35,000	500	0.0383	0.0143	-1.28	0.29
1898	12,700	1,850	1,300	650	0.1457	0.5000	-1.05	2.28
1899	8,700	330	1,500	90	0.0379	0.0600	-1.30	1.76
1899	55,000	30	13,500	910	0.0005	0.0674	3.41	1.40
1899	10,200	500	6,000	20	0.0490	0.0033	-3.22	0.53
1899	3,400	260	1,200	90	0.0765	0.0750	-1.06	1.04
1899	4,500	230	1,500	110	0.0511	0.0733	-0.74	1.10
1899	25,000	180	9,500	320	0.0072	0.0337	0.58	0.97
1899	450	200	5,000	760	0.4444	0.1520	1.34	-2.41
1899	4,000	120	11,500	900	0.0300	0.0783	2.01	-1.06
1899	8,500	200	2,500	40	0.0235	0.0160	-1.61	1.22
1900	12,000	430	2,000	400	0.0358	0.2000	-0.07	1.79
1900	1,500	20	1,200	160	0.0133	0.1333	2.08	0.22
1900	5,000	500	12,000	1,110	0.1000	0.0925	0.80	-0.88
1904	145,000	17,000	210,000	46,000	0.1172	0.2190	1.00	-0.37
1904	140,000	75,000	40,000	27,500	0.5357	0.6875	-1.00	1.25
1904	135,000	17,500	150,000	16,500	0.1296	0.1100	-0.06	-0.11
1904	40,000	4,600	13,000	900	0.1150	0.0692	-1.63	1.12
1904	36,000	1,200	38,000	3,000	0.0333	0.0789	0.92	-0.05
1904	24,000	850	12,000	1,550	0.0354	0.1292	0.60	0.69
1904	36,000	1,100	8,000	1,800	0.0306	0.3000	0.49	1.79
1904	22,000	1,250	24,000	2,000	0.0568	0.0833	0.47	-0.09
1905	40,000	9,400	58,000	13,000	0.2350	0.2241	0.32	-0.37
1905	314,000	41,000	310,000	71,000	0.1306	0.2290	0.55	0.01

Endnotes

1. James Schneider, "The Exponential Decay of Armies in Battle," Theoretical Paper No.1, (Fort Leavenworth, KS: School of Advanced Military Studies, 1985).

2. Ronald L. Johnson, "Lanchester's Square Law in Theory and Practice," Monograph, (Fort Leavenworth, KS: School of Advanced Military Studies, 1990).

3. David G. Chandler. The Campaigns of Napoleon, (New York:MacMillan Publishing Company, Inc., 1986), p.66-67, 454.

4. James G. Taylor, Force-On-Force Attrition Modelling, (Arlington, Va: Operations Research Society of America, 1980), p. 5.

5. This information was extracted from a TRADOC Analysis Command- Fort Leavenworth handout developed by TRAC-FT LVN Training Simulations Directorate, Ft. Leavenworth, KS, 1989.

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7. Trevor N. Dupuy, Numbers, Prediction and War, (Fairfax, Va:HERO Books, 1985), p. 4.

8. "Commentary on Frederick William Lanchester" in The World of Mathematics, IV , J. Newman, editor, (New York: Simon and Schuster, 1956), p. 2136.

9. Frederick W. Lanchester, "Mathematics in Warfare", Engineering 98, (1914), p. 422-423 (re-printed in The World of Mathematics, IV, (New York: Simon and Schuster, 1956), p. 2144.

10. Ibid, p. 2139

11. Ibid, p.2139.

12. Taylor, p. 31. Note that the equations throughout the paper will use variables R and B to represent the opposing forces in order to maintain consistency between the different models.

13. Ladislov Dolansky, "Present State of Lanchester Theory of Combat," Operations Research Vol 12, (Mar-Apr 1964), p. 345.

14. Taylor, p. 23.

15. Johnson, p. 8-9.

16. Lanchester, p. 2147.

17. Taylor, p. 31.

18. Dolansky, p. 345.

19. Taylor, p. 24.

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